Fourier Analysis

Handin question 7

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1. (a) A dynamical system has a response, $y(t)$, to a driving force, $f(t)$, that satisfies a differential equation involving a third time derivative:

$$t^2\, d^3y/dt^3 = f(t).$$

Obtain the solution to the homogeneous equation, and use this to derive the causal Green's function for this system, $G(t,\tau)$. **[7]**

(b) We want to solve the general linear 2nd-order differential equation

$$\frac{d^2y}{dt^2} + p(t)\frac{dy}{dt} + q(t)y = f(t),$$

subject to the boundary conditions $y(a) = y(b) = 0$. If $y_1(t)$ and $y_2(t)$ are independent solutions of the homogeneous equation, show that these can be chosen so that each solves one of the boundary conditions – i.e. $y_1(a) = y_2(b) = 0$. **[4]**

(c) Neglecting the possibility that either y_1 or y_2 solves both boundary conditions (which can happen in special cases), show that the Green's function for the interval $a < (t,T) < b$ is

$$G(t,T) = \begin{cases} y_1(t)y_2(T)/W(T) & a < t < T \\ y_1(T)y_2(t)/W(T) & T < t < b \end{cases},$$

where $W(t) = y_1(t)\dot{y}_2(t) - \dot{y}_1(t)y_2(t)$ is the Wronskian. **[4]**

(d) Hence obtain the solution of

$$\frac{d^2y}{dt^2} = \begin{cases} 0 & 0 < t < c \\ 1 & c < t < 1 \end{cases},$$

with the boundary conditions $y(0) = y(1) = 0$. The cases $t < c$ and $t > c$ should be considered separately. **[10]**