Fourier Analysis

Handin question 5

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1. A one-dimensional harmonic oscillator with natural frequency ω_0 is driven with a driving acceleration $a(t)$, so obeys

$$\frac{d^2z}{dt^2} + \omega_0^2 z = a(t).$$

(a) Take the Fourier Transform of this equation (from t to ω) and show that

$$\tilde{z}(\omega) = \frac{\tilde{a}(\omega)}{\omega_0^2 - \omega^2}$$

[5]

(b) Hence show that

$$z(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\tilde{a}(\omega)}{\omega_0^2 - \omega^2} e^{i\omega t} d\omega.$$

[3]

(c) If $a(t) = \sin^2 \Omega t$, find $\tilde{a}(\omega)$ [hint: do not use a Fourier series. Stay within the framework of Fourier transforms]. **[10]**

(d) Hence find a solution for $z(t)$ (ignore solutions to the homogeneous equation). **[7]**