Fourier Analysis

Handin question 3

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Session: 2014/15
Handin Deadline: 4 p.m. Thursday 16th October 2013

1. (a) Consider the function

$$f(x) = \begin{cases} x & x < 1 \\ x - 2 & x > 1. \end{cases}$$

Write $f(x)$ in terms of the Heaviside function and hence give the derivative of f. **[4]**

(b) Give the general definition of $\tilde{f}(k)$, the Fourier transform of a function $f(x)$, and write down the transform of the Dirac delta-function, $f(x) = \delta(x-c)$. By considering $\tilde{f}(k)\exp(iky)$ and using a delta-function, derive the general relation for the inverse Fourier transform. **[6]**

(c) Similarly, prove that repeating the Fourier transform yields almost the original function f:

$$\widetilde{\tilde{f}}(K) = 2\pi f(-K).$$

What happens if we transform the complex conjugate of $\tilde{f}$? **[5]**

(d) Compute the inverse Fourier transform of a delta-function in k space:

$$\tilde{f}(k) = \delta(k - K).$$

[4]

(e) Using this result, write down the Fourier transforms of $\sin(\alpha x)$ and $\cos(\beta x)$. Hence show that a periodic function written as a Fourier series can also be expressed using Fourier transforms. **[6]**