# Cosmological Physics: additional topics

## 1.1 The event horizon in a vacuum-dominated universe

If the universe contains a cosmological constant, the future is one of accelerating expansion as our model tends to approach de Sitter space. This model contains an event horizon, unlike models without vacuum energy. Where does this leave the eventual visibility of parts of the universe that we can see today? Consider a flat model, for which the comoving distance to the event horizon is

$$D_{\rm EH} \equiv R_0 r_{\rm EH} = \frac{c}{H_0}\int_{-1}^{0} \frac{dz}{\sqrt{1-\Omega_m+\Omega_m(1+z)^3}} = \frac{c}{H_0}\left[1+3\Omega_m/8+O(\Omega_m^2)\right]. \qquad (1.1)$$

Note that this is identical to the integral for the particle horizon, except that the redshift limits would be 0 to $\infty$ in that case.

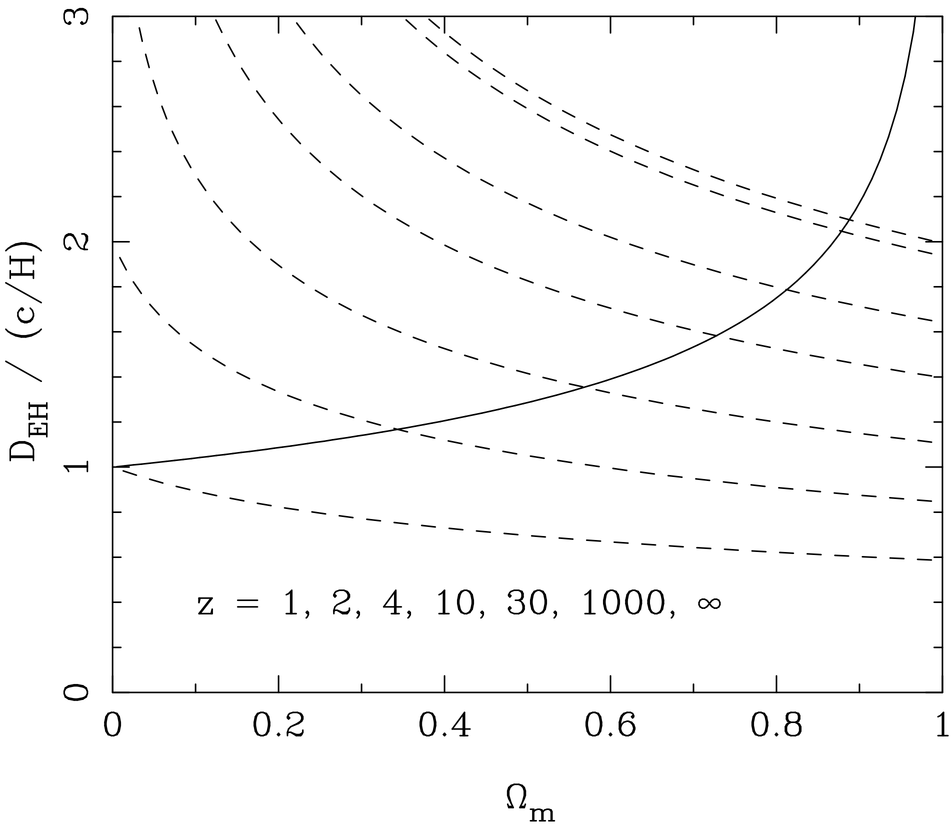


**Figure 1.1.** A plot of the comoving distance to the event horizon in various flat models (solid line). The dashed lines show the comoving distance to various redshifts. These show that, for $\Omega_m < 0.88$, we have already lost causal contact with regions near the particle horizon, such as last scattering. At present, we are on the point of losing causal contact with lower-redshift regions ($z = 1.8$ for $\Omega_m = 0.3$).

This expression for $D_{\rm EH}$ is plotted in figure 1.1, and contrasted with the comoving distance to different redshifts. If $\Omega_m$ is close to zero, the model is essentially de Sitter, and the event horizon corresponds to the distance to redshift 1. For significant values of $\Omega_m$, the event horizon is larger, and for $\Omega_m = 0.88$ it reaches all the way to the current particle horizon. For $\Omega_m = 0.3$, the limit is $z = 1.8$. Thus, a good fraction of the faint galaxies now seen are sufficiently distant that it is impossible even in principle to visit them by rocket. Certainly, it is far too late to plan a trip to the last scattering sphere.

Things will become still worse in the future, as the event horizon settles down to a constant proper size, and more and more of its contents are swept outside by the expansion. Figure 1.1 can be used to analyse this situation immediately: we just need to interpret $H$ and $\Omega_m$ as the values at the era of interest:

$$\begin{aligned} H^2(a) &= H_0^2 \left(1 - \Omega_m + \Omega_m a^{-3}\right) \\ \Omega_m(a) &= \frac{\Omega_m}{\Omega_m + (1 - \Omega_m)a^3} \end{aligned} \tag{1.2}$$

(for a flat model). We then obtain the proper size of the event horizon, which tends to

$$D_{\rm EH}^{\infty} = \frac{c}{H(\infty)} = (1 - \Omega_m)^{-1/2} \frac{c}{H_0}. \tag{1.3}$$

For $\Omega_m = 0.3$, this is only 6% larger than its current value.

At a future time when the scale factor is $a$, any current comoving distance will expand by a factor $a$ and will pass outside $D_{\rm EH}$ for large enough $a$. For example, the limit of homogeneity in the universe, $r = 100\,h^{-1}$ Mpc, will be lost to causal contact when $a = 36$. We can also ask how things were in the past, when the comoving value of $D_{\rm EH}$ was larger. For $\Omega_m = 0.3$, $D_{\rm EH}$ was equal to the distance to redshift 1000 at a time corresponding to $z = 7.4$: this was the last chance to send a probe to see what became of the material that emitted our microwave background.