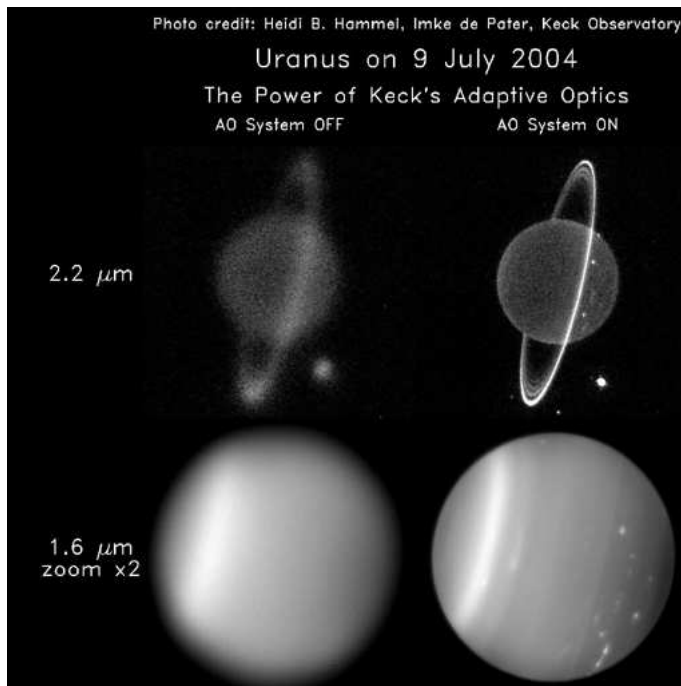


# Atmospheric Turbulence and its Influence on Adaptive Optics

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# 1 Introduction

Adaptive Optics (AO) is still a relatively young branch of astronomical instrumentation. Different technologies are developing all the time and it can be difficult keeping up with all the different acronyms (e.g. GLAO, MOAO, MCAO, SLGS, RLGS, etc..) never mind the technologies themselves. It is easy to question why we need all these various types of AO systems. The answer is unsurprisingly that there is no one AO system appropriate for all conditions. The AO system is dependent on the instruments that will use it, the feasibility of the AO system at the telescope, the atmospheric conditions at the telescope site, and the budget available. In this lecture I wish to address the atmospheric parameters that need to be considered when deciding on an AO system and describe which AO systems are best suited for which conditions. Time permitting, I will then describe some of the instruments used to measure the atmospheric turbulence profile.

There are a few equations throughout the talk, however they are mainly here to show the dependence of the different terms I will discuss. I will not derive any of them as most of the derivations are long, and most of the lecture would be taken up getting between equation (2) and (5) and that would be productive for neither of us.

Light arriving from a star enters the atmosphere as a plane wave. Differential temperatures in the atmosphere ( $< 1^\circ\text{C}$ ) cause random changes in wind velocities (eddies), which we observe as turbulence. The temperature variations result in small changes in atmospheric density, hence changing the atmospheric refractive index. The varying refractive index causes the wavefront to change phase as it propagates through to the detectors, leading to beam wander and beam spread. The varying phases start to interfere resulting in a variation in intensity (scintillation). Beam wander is referred to as tilt and is caused by eddies that are large with respect to the beam of light. Eddies that are smaller than the beam cause it to break up as different sections of the beam are refracted in different directions. When long exposures are taken, the image is blurred which effectively makes the image Point Spread Function (PSF) broaden from the diffraction limited case.

The electromagnetic wave after it has been distorted by the atmosphere can be represented in terms of amplitude and phase

$$A(\rho, \theta) \exp[-i\Phi(\rho, \theta)] \quad (1)$$

The principle of adaptive optics is to remove the distortions created by the atmosphere. To apply a change which flattens the phase we multiply by the complex conjugate  $\exp[+i\Phi(\rho, \theta)]$ . In adaptive optics terms, the deformable mirror acts as the complex conjugate.

## 2 Atmospheric Turbulence

The atmosphere can be visualised as a path of continually changing lenses refracting light as it travels towards the telescope. The lenses are created by turbulent eddies created at various different altitudes. Tatarskii (1961) made the assumption that the variations in refractive index result directly in phase fluctuations described by  $\phi(\mathbf{r})$ , whereas any change in amplitude are due to second order effects acting on the already perturbed wavefront. This turns out to be a reasonable assumption as most models of the Earth's atmosphere in the optical and infra-red show that imaging performance is dominated by phase fluctuations and that the amplitude fluctuations have a negligible effect on the structure of images seen in the focus of a large telescope. It is possible to consider the atmospherically induced variance between the phase at two parts of the wavefront separated by distance  $\mathbf{r}$  by assuming a structure function. A structure function is the average difference between two values of a random process. Tatarskii assumed the random phase variations were Gaussian, giving the structure function the form

$$D_{\phi_a(r)} = \langle [\phi_a(\mathbf{r}_1 + \mathbf{r}) - \phi_a(\mathbf{r}_1)]^2 \rangle \quad (2)$$

Where the phase is represented by

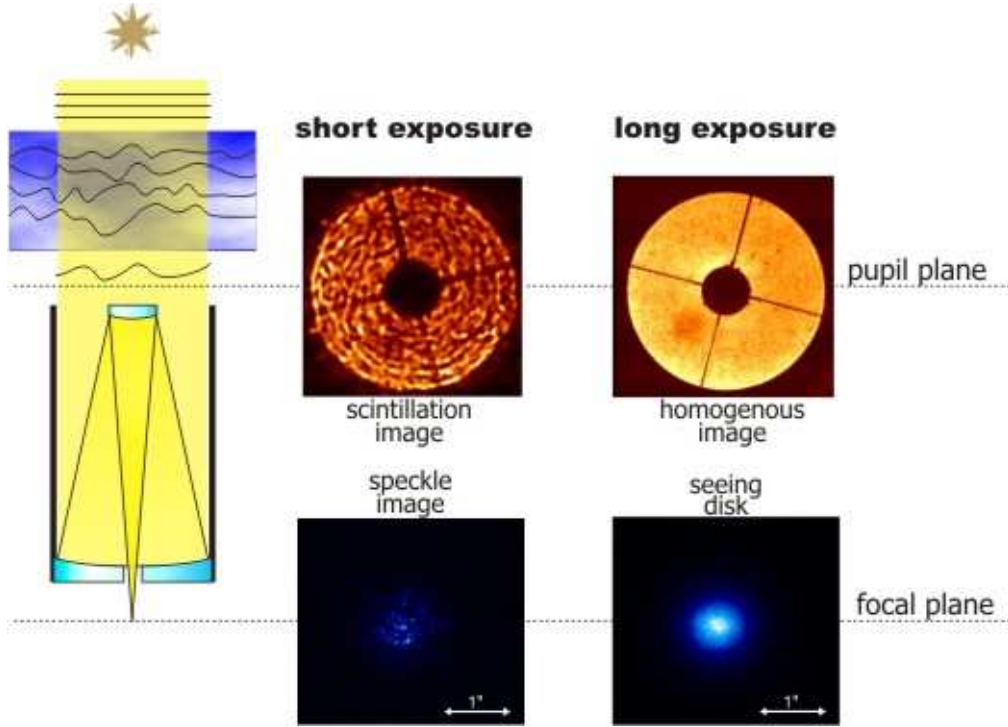


Figure 1: A diagram depicting the effects of atmospheric turbulence. The scintillation pattern is from the pupil plane and the speckle image is from the focal plane. The short exposures are  $\sim 1$  msec, whereas long exposures are  $> 1$  sec. Courtesy of Sebastian Egner; [http://www.mpia.de/homes/egner/documents/scidar.html#prin\\_scidar](http://www.mpia.de/homes/egner/documents/scidar.html#prin_scidar)

$$\phi(\mathbf{r}) = \frac{2\pi}{\lambda} l(\mathbf{x}) \quad (3)$$

$l(\mathbf{x})$  represents the optical path. Whilst air is slightly dispersive, its effect is negligible, hence the perturbations of the optical path length are seen as achromatic. Therefore, the wavelength dependency of the structure function is:

$$D_{\phi_a(r)} \propto \lambda^{-2} \quad (4)$$

The Kolmogorov model of turbulence makes three assumptions about the atmosphere. First, the atmosphere is locally homogeneous (velocity difference between points in space depends on the separation between them). Second, the atmosphere is isotropic (the velocity difference is only dependent on the magnitude of the separation, not the direction), and thirdly, the turbulence is incompressible. Tatarskii used the Kolmogorov model of turbulence to define the structure function in terms of only a single parameter, the atmospheric coherence length, or the Fried parameter  $r_0$ .

$$D_{\phi_a(r)} = 6.88 \left( \left| \frac{\mathbf{r}}{r_0} \right| \right)^{5/3} \quad (5)$$

The atmospheric coherence length is an extremely useful parameter when considering atmospheric turbulence. Noll showed the variance of the wavefront phase averaged over the aperture approximates to unity.

$$\sigma^2 = 1.02 \left( \frac{r}{r_0} \right)^{5/3} \quad (6)$$

So the atmospheric coherence parameter is the length over which optical phase distortion has mean square value of  $1 \text{ rad}^2$ . Waves smaller than  $r_o$  can travel through the atmosphere and arrive at the detector still coherent. Therefore, if your telescope is of diameter smaller than  $r_o$ , then you will receive an image of limited phase distortion and the object PSF will be limited to the diffraction limit of the telescope i.e.  $1.22 \frac{\lambda}{D}$ . Telescopes of diameter larger than  $r_o$  will still only have the resolution of a telescope of diameter  $r_o$ . Since the path length is assumed achromatic,  $r_o \propto \lambda^{\frac{6}{5}}$ .

The atmospheric coherence length is often used for describing the level of turbulence at a site. It is dependent on the refractive index structure constant  $C_n^2$ , which is variable on both long and short time-scales, seasonal to seconds and is clearly not constant. It is dependent on wind strength and so is highly variable with altitude and zenith angle. It is also variable from site to site, and it is this profile as a function of altitude which is studied when considering different sites for a new telescope. The relation between  $C_n^2$  and  $r_o$  is given by

$$r_o = \left[ 0.423 \left( \frac{2\pi}{\lambda} \right)^2 \sec(z) \int_0^L C_n^2(h) dh \right]^{-\frac{3}{5}} \quad (7)$$

Where  $z$  is the zenith angle,  $L$  is the path length i.e. the distance to the top layer of turbulence. The refractive index structure constant can be used to calculate the seeing conditions at a site. For this we need to consider the imaging equation.

## Seeing

An image can be described by its spectral components, i.e. the large light and dark areas, and the fine detail. How much of this information is transferred to the image is dependent on the contrast, or the variations between the dark and light aspects of the image, which is referred to as the modulation. An imaging system, in our case a telescope, transfers the modulation from the object to the image. The mathematical form the modulation is the modulation transfer function (MTF). If the image is transferred perfectly,  $MTF = 1$ , if the contrast is lost completely  $MTF = 0$ , and a blurry image is within 0 and 1. The MTF is the modulus of the Optical Transfer Function (OTF).

The OTF can be broken down into phase and magnitude components. The OTF of a telescope comprises the OTF of both the atmosphere and the telescope optics. However, since modern telescopes contain such high quality optics, the OTF can be considered to come solely from the atmosphere. The atmospheric OTF is related directly to the atmospheric phase structure function.

$$OTF_a(\mathbf{f}) = \exp[-0.5D_\phi(\lambda\mathbf{f})] \quad (8)$$

The PSF of the image is the FT of the OTF, hence if we know the OFT, we can describe the image PSF. Since we know the OTF in terms of  $r_o$ , it is possible to directly relate the FWHM of the PSF to the atmospheric coherence length.

$$\beta = 0.98 \frac{\lambda}{r_o} \quad (9)$$

$\beta$  is the FWHM of the image PSF and is often referred to as the seeing. Hence, the FWHM of the PSF is dependent on the atmospheric coherence length rather than the diameter of the telescope which was the case for the diffraction limited telescope. Since  $r_o \propto \lambda^{\frac{6}{5}}$  we can see  $\beta \propto \lambda^{-\frac{1}{5}}$  confirming that the seeing conditions in the infra-red are better than in the optical!

Since  $r_o$  states the distance over which phase fluctuations become important, it is this parameter which defines the maximum distance that actuators on the deformable mirror (DM) can be separated by. If separated by more than this then the DM will not be able to fully correct the phase distortions. More importantly,  $r_o$  determines how effective an AO system will be at making corrections, or for the most part, partial corrections.

### 3 Anisoplanatism

Often is the case with AO that the exact turbulence experienced by the science source is not easily measurable. The variation between the measured turbulence and that experienced by the science source can be found in terms of  $r_o$ .

Atmospheric turbulence varies at different altitudes and zenith angles, so the turbulence in any line of sight will differ to the next. If the aberrations were uniform in each direction, then the atmosphere could be considered isoplanatic. Unfortunately this is not the case, the atmosphere is anisoplanatic. AO systems use light from bright objects to measure phase fluctuations. They are referred to as Guide Stars (GS) and can be naturally occurring (i.e. bright stars within the Field of View (FoV), or artificially created ones). There are various different forms of anisoplanatism, all arising from the fact that the GS are not measuring the exact turbulence path as the science source.

#### Angular Anisoplanatism

Angular anisoplanatism occurs when the sensor is fixed in one position and the GS and science object are separated. Good corrections can still be made when the offset between the objects is small. The angle at which effective corrections can still be made is given by the isoplanatic angle  $\theta_0$ , again derived from the Kolmogorov model of turbulence.

$$\theta_0 = \left[ 2.91 \left( \frac{2\pi}{\lambda} \right)^2 \sec^{\frac{8}{3}} z \int_0^L C_n^2(h) h^{\frac{5}{3}} dh \right]^{-\frac{3}{5}} \quad (10)$$

This simplifies when  $C_n^2$  is constant along a path of length L to

$$\theta_0 \approx 0.6 \left( \frac{r_o}{L} \right) \quad (11)$$

The wavefront variance associated with the angle between the GS and the science object can be represented by

$$\sigma_{iso}^2 = \left( \frac{\theta}{\theta_0} \right)^{\frac{5}{3}} \quad (12)$$

If the angular separation between the objects is smaller than  $\theta_0$  then the adaptive optics correction can be very good. If on the other hand the angle is much larger than  $\theta_0$ , then correction will not be good and in fact can do more harm than good.

#### Temporal Anisoplanatism

Since the atmosphere is always mixing, the phase distortions are always changing. It is important that an AO system can measure the phase variations and make the corrections before the atmosphere changes and new corrections are required. As previously stated, different size eddies cause different imaging effects. The larger eddies cause tilt variations and these vary on longer time scales than the higher-order turbulent layers. If corrections are not made before the atmosphere moves, then the tilt will cause the image to be smeared. For high-order turbulent layers the corrections should be made on timescales which keep the residual wavefront variance small. An AO system should correct for most of the phase fluctuations. By assuming the Kolmogorov model of turbulence, Greenwood calculated the characteristic frequency  $f_G$  of the atmosphere, referred to as the Greenwood frequency.

$$f_G = 2.31 \lambda^{-\frac{6}{5}} \left[ \sec(z) \int_0^L C_n^2(h) v_w^{\frac{5}{3}}(h) dh \right]^{\frac{3}{5}} \quad (13)$$

Where  $v_w$  refers to the wind speed. When it is possible to assume a constant wind velocity and knowing the path integrated  $r_o$ ,  $f_G$  reduces to:

$$f_G = 0.43 \frac{v_w}{r_0} \quad (14)$$

The residual wavefront variance associated with the response of the AO system to  $f_G$  can be represented by

$$\sigma_{temp}^2 = \left( \frac{f_G}{f_{AO}} \right)^{\frac{5}{3}} \quad (15)$$

Where  $f_{AO}$  refers to the frequency of the AO system. So the frequency at which corrections need to be made is dependent on the atmospheric coherence length and the velocity of the winds

## Focal Anisoplanatism

Focal anisoplanatism occurs when using artificial GS. Artificial GS are created by stimulating a layer in the upper atmosphere with a laser creating a ‘‘beacon’’. There are two different forms of LGS, Sodium beacon LGS (SLGS) and Rayleigh beacon LGS (RLGS). The details of these could cover a whole lecture course in their own right, so I will only make a passing comment on them. Sodium atoms are stimulated high in the mesosphere ( $\sim 90\text{km}$ ) with a laser specially tuned to  $589.2\text{nm}$ . They then re-emit producing a glowing artificial star. The Rayleigh beacons rely on scattering of light by molecules lower in the atmosphere. Often the light is pulsed so that only light scattered higher in the atmosphere is detected. RLGS are less expensive, but generate GS lower in the atmosphere ( $\sim 20\text{km}$ ).

Focal anisoplanatism is also referred to as the *cone effect*. LGS only sample a cone of the turbulence seen by the science source, hence the phase fluctuations can only be partially corrected. The variance between the LGS phase fluctuations science source fluctuations was derived by Fried.

$$\sigma_{cone}^2 = \left( \frac{D}{d_0} \right)^{\frac{5}{3}} \quad (16)$$

Where  $D$  is the telescope aperture and  $d_0$  is the focal anisoplanatic parameter (in meters) derived by Tyler.

$$d_0 = \lambda^{\frac{6}{5}} \cos^{\frac{3}{5}}(z) \left[ 19.77 \int \left( \frac{h}{h_{LGS}} \right)^{\frac{5}{3}} C_n^2(h) dh \right]^{-\frac{3}{5}} \quad (17)$$

Where  $h_{LGS}$  is the altitude of the LGS (in  $km$ ). The point to notice is the dependence on the altitude of the LGS. The higher they are stimulated, the lower the variance between the turbulence. The cone effect can be overcome somewhat by using multiple GS to assess the turbulence.

## 4 $C_n^2$ Profiles

AO systems vary in complexity, cost and feasibility for every telescope. The anisoplanatic terms mentioned previously are all dependent on the atmospheric coherence length. Sites with a large  $r_0$  value will provide the best uncorrected observations. However, it is when we analyse the constituent layers of the atmosphere that we discover how effective an AO system will be. The form of the  $C_n^2$  profile decides what AO system is most appropriate for a particular site.

Figure 2 contains 4 different  $C_n^2$  profiles taken on different nights. They are taken from the VLT ground layer turbulence profiler. The top graph in each image shows the integrated turbulence strength above  $1500\text{m}$ . The lower graphs show the variation of  $C_n^2$  with altitude and time. Each altitude bin varies with time as the elevation of the target stars vary. These graphs illustrate how variable a  $C_n^2$  profile can be from night to night.

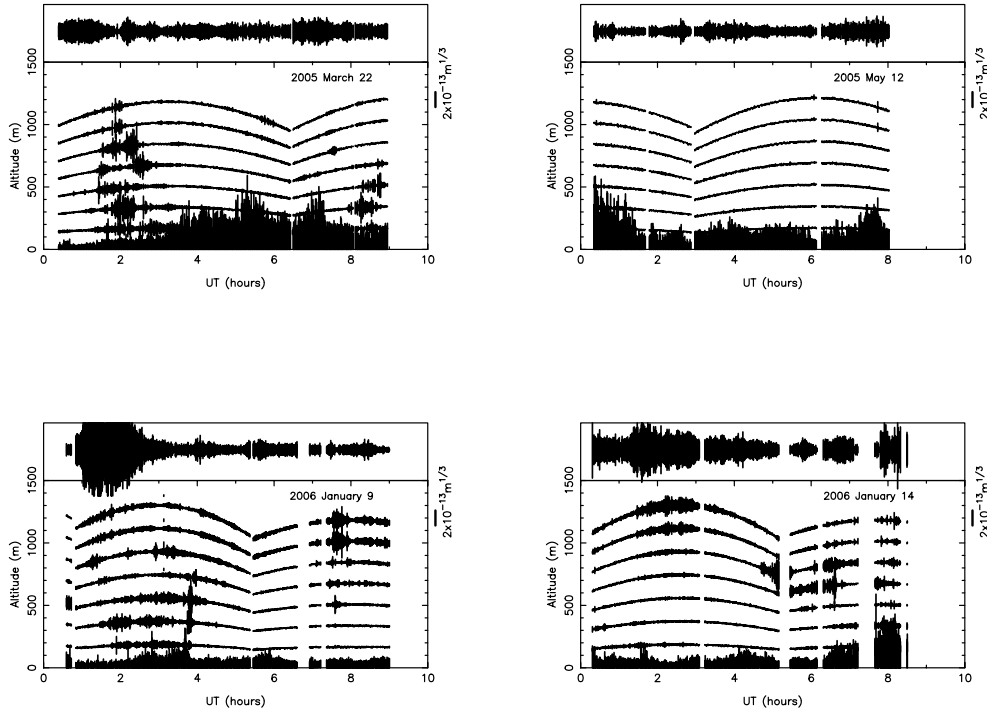


Figure 2: Four  $C_n^2$  profiles taken with ground-layer turbulence profiler at the VLC. Top plots on each graph shows integrated turbulence strength above 1500m. Courtesy of Richard Wilson, Astronomical Instrumentation Group, Durham

Hundreds of these profiles are collected per site. The median  $r_0$  is found for all the images and all the  $C_n^2$  profiles with  $r_0$  near the median value are combined to find the average profile. Figure 3 gives an example for Cerro Pachon using SCIDAR.

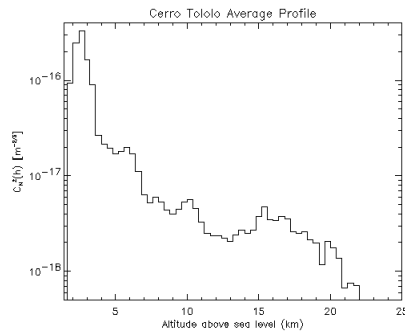


Figure 3: Average  $C_n^2$  profile for Cerro Pachon. Courtesy of J. Vernin, University of Nice

The layers of turbulence that most strongly effect the value of  $r_0$  occur in the lower atmosphere. In fact, at some sites almost all of the contribution comes from 0-20km up (as in the Cerro Tololo example). When this is the case, correcting only the low altitude turbulence may be sufficient in providing high



quality images with low FWHMs. In this case it would seem appropriate to employ Ground Layer Adaptive Optics (GLAO). Most of the turbulence can be corrected for with a simpler and cheaper technology than would be required for correcting the whole atmosphere.

In cases where high altitude turbulence is important, the use of NGS would be preferable. Unfortunately there are few NGS bright enough per FoV that can be of use. Even when there are, it is likely that the separation from the science object is large resulting in angular anisoplanatism. Turbulence in the higher atmosphere also reduces the isoplanatic angle increasing the problem. In this scenario LGS would be a better option. SLGS can be excited near the science object, reducing the angular wavefront variance,  $\sigma_{iso}^2$ . This is useful if you wish to observe one particular object within the FoV, however often it is the whole FoV of interest. The further away for the SLGS, the worse the correction becomes. Multiple GS would be appropriate for this case. Two such forms of adaptive optics are Multi-Conjugate Adaptive Optics (MCAO) and Multi-Object Adaptive Optics (MOAO).

## 5 Measuring Atmospheric Turbulence

So how do we measure  $C_n^2$  profiles in order to determine which AO system we should use? There are various different techniques for measuring wavefront errors. The techniques are used for both site characterisation, and real-time AO corrections. Here I will address a few of the more common methods employed.

### DIMM - Differential Image Motion Monitor

DIMM instruments measure the differential centroid motion of two images resulting from one star split across two apertures of known separation. A quick frame rate allows many images to be taken per second, recording the image motions. The variance of the differential image motions can be found parallel and perpendicular to the aperture alignments. The differential aspect of the device means that the measurements are insensitive to telescope tracking errors, vibrations, or small focus errors. Therefore, the differential motion variance is caused only by the atmospheric turbulence and can be directly related to  $r_0$  using Tatarskii's model of turbulence. DIMM instruments can only measure the seeing conditions, they are not sensitive to altitude, so cannot record  $C_n^2$  profiles. They are extremely common instruments which most observers will be aware of as they are used to give an idea of how the conditions were when their observations were taken.

### Shack-Hartmann wavefront sensor

A Shack-Hartmann wavefront sensor is an extremely common wavefront sensor used for measuring phase aberrations. A wavefront travels through an array of lenses (lenslets) all with the same focal length. The beam is focused into small sub-apertures (on the same detector). A plane wave will create a grid of equidistant focal spots. For a distorted wave, the phase fluctuations at each section of the beam can be seen as a discrete set of tilts, moving the positions of the focal spots. Thus, a Shack-Hartmann wavefront sensor measures the wavefront slopes which, when reconstructed, forms the whole wavefront. The reconstruction is a standard procedure well within the capabilities of modern computers. The reconstruction is based on Zernike polynomials. (This is not the time or the place for an in depth discussion on Zernike polynomials however, more information can be found by Bezdiko, S. N. "The Use of Zernike Polynomials in Optics." Sov. J. Opt. Techn. 41, 425, 1974. Or if you wish to have a graphical play varying to see which polynomial recreates which aberration - <http://wyant.optics.arizona.edu/zernikes/zernikes.htm>. The resolution of the wavefront is the size of each sub aperture. Figure 4 shows the principles of the Shack-Hartmann device.

### SCIDAR - SCIntillations Detection And Ranging

Classical SCIDAR works by imaging two binary stars with an angular separation of  $\alpha$ . If we consider the simplest case, where there is one turbulent atmospheric layer at an altitude of  $h$ , then two scintillation

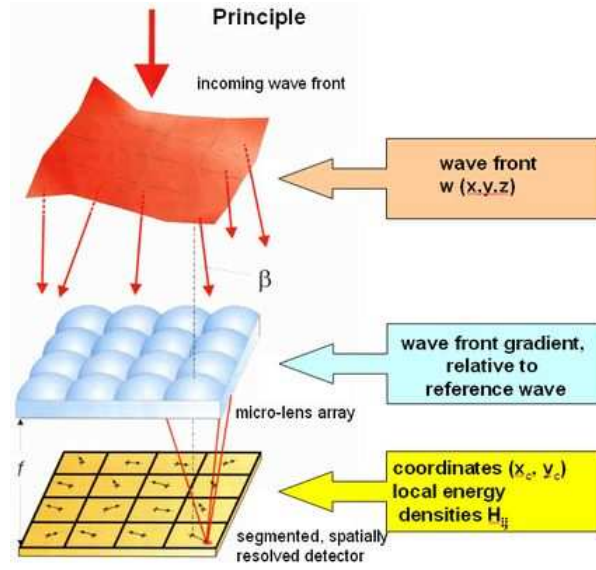


Figure 4: A diagram of the principles of a Shack-Hartmann wavefront sensor

patterns are created, separated by a distance  $d = \alpha h$ . The autocorrelation function will find the repeating separation pattern between the binary stars over a number of images. The separation in peaks from the averaged autocorrelation function will yield  $d$ , hence the height of the turbulent layer can be found.

The velocity of the turbulence can also be measured by looking at the variations in the scintillation patterns from image to image. Each image lasts for a period of  $\Delta t$ . The patterns will move a distance  $v_w \Delta t$  from the correlation centre from one image to the next. Since  $\Delta t$  is a known parameter, the wind velocity  $v_w$  is easily deduced. Figure 5 provides a schematic of how SCIDAR works. SCIDAR requires a telescope  $>1m$  so it is not really a portable system.

Generalised SCIDAR works in the exact same manner, only from  $2km$  beneath the telescope focal plane. This is because the variance of scintillation follows  $\sigma_{scint} \propto h^{\frac{5}{6}}$ , hence the effect of scintillation is undetectable at a distance of  $2km$  above the detector.

(For more information on SCIDAR - Vernin & Muñoz-Tuñi (1994), Klückers et al (1998), Avila, Vernin & Sánchez (2001))

## SLODAR - SLOpe Detection And Ranging

SLODAR works in a similar manner to SCIDAR, only the local *gradient* of the phase aberrations are measured using a Shack-Hartmann wavefront sensor. Again a binary star system is used with separation  $\alpha$ . The two images travel through the lenslet, creating two focal spots on each sub-aperture. The binary star must have a separation large enough that they are resolved in each sub-aperture. The sub-apertures must also be large enough to allow measurement of centroid motions due to the seeing. Essentially each sub-aperture is measuring a different turbulence layer of thickness  $\delta h$

$$\delta h = \frac{(D/n_{sub})}{\alpha} \quad (18)$$

From this, you can calculate to what altitude it's possible to measure the atmospheric turbulence simply through  $h_{max} = n_{sub} \delta h$ . As can be seen, the larger the separation in the binary star, the better resolution of atmospheric height, but at the cost maximum height you can observe.

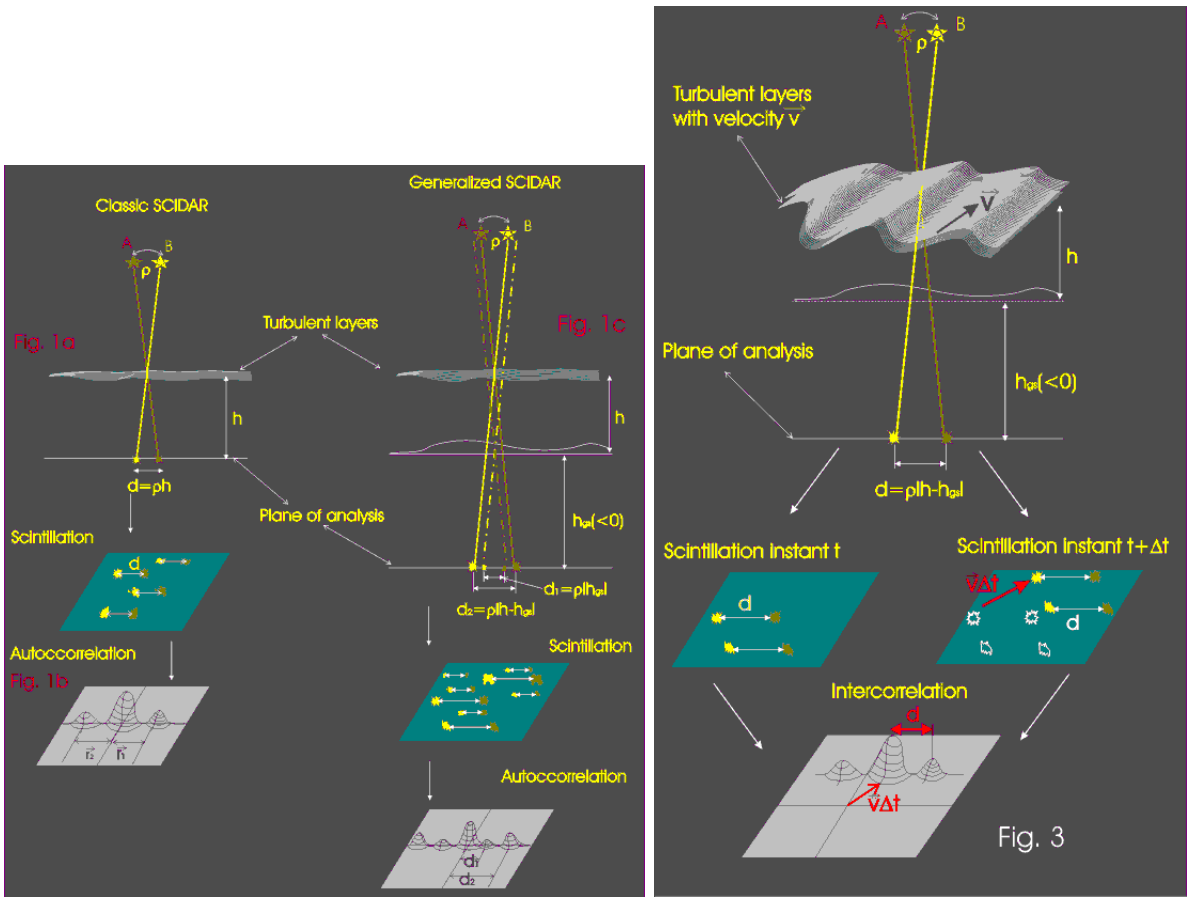


Figure 5: Diagrams explaining the principles of SCIDAR. Courtesy of the Instituto de Astronomia, Universidad Nacional Autonoma de Mexico (UNAM)

The wind velocities of individual turbulent layers can be found in the same manner as used for SCIDAR using the non-zero time delays between images. (for more information on SLODAR - R. W. Wilson, SLODAR: measuring optical turbulence with a Shack-Hartmann wavefront sensor (2002))

### MASS - Mult-Aperature Scintillation Sensor

MASS also derives the turbulence profiles from scintillation patterns, however it does not rely on binary stars. The spatial fluctuations in intensity are dependent on the altitude of the turbulence causing them. Using this premise, A. Tokovinin devised a system of concentric ringed apertures (photon counters in this case) to separate the contributions from different layers. For each ring a 'scintillation index' is computed which is the intensity variance normalised by the square of the average intensity. Using the scintillation index instead of intensity variance removes any dependence on the brightness of the measured star and reflects only the strength of the atmospheric scintillation. Photon noise obviously has been accounted for and removed at this part of the calculation. Similarly, the differential scintillation index is also computed for a pair of apertures. This is defined as the variation of the ratio of intensities from one aperture to the next, and again is normalised by the square of the average intensity ratio. The indices of a layer are then used to compute the turbulence intensity as a weighting function which is dependent on the distance to the layer as well as the shape and size of the apertures. So essentially it finds the weighted strength of turbulent at altitude. A MASS system is then usually combined with DIMM so that the physical turbulence profile can be found.

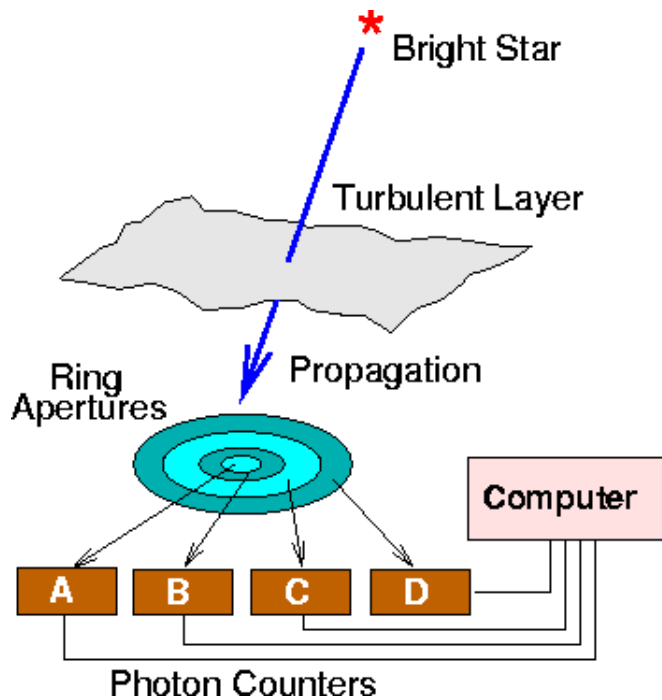


Figure 6: A basic description of MASS. Courtesy of A. Tokovinin <http://www.ctio.noao.edu/atokovin/profiler/>

## 6 Summary

Whilst the atmosphere limits the performance of a telescope, it also plays an important role in the determination of which AO system would be most efficient for a particular instrument. There are various instruments available to determine the main atmospheric characteristics, such as atmospheric coherence length, atmospheric coherence time,  $C_n^2$  profile, etc.. From these terms it is possible to predict the how accurately the AO system will be at making phase corrections, hence will inform of the expected FWHM of the PSF achievable. Taking these factors into consideration, along with budget, time, and other constraints, can allow for a rational method on deciding which AO system is correct for you.

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