Understanding the Cosmic Microwave Background Temperature Power Spectrum

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The Cosmic Microwave Background

The Cosmic Microwave Background (CMB) radiation field is an open window to the early Universe. It is a nearly-uniform and isotropic radiation field, which exhibits a measured perfect black-body spectrum at a temperature of 2.72K. This primordial radiation field is a prediction from a Big Bang universe - if in its early stages the Universe was at a high enough temperature to be fully ionised then processes such as Thompson scattering and Bremsstrahlung would thermalise the radiation field very efficiently. Assuming an adiabatic expansion of the Universe, one would then expect to observe a radiation field which would have retained the black-body spectrum, but at a much lower temperature.

As observers, we can measure three things about this radiation: its frequency spectrum $f(\nu)$, its temperature $T(\hat{\mathbf{n}})$ and its polarisation states. Each of these observables contains information about the creation and evolution of the field and are fully packed with cosmological information. Although the study of the polarisation of the CMB radiation has been a recent and promising area of research (propelled by technology advancements which now allow this signal to be measured), this lecture will concentrate mostly on the temperature signal. Even so, the subject is vast and a full treatment would require many advanced lectures. In this one lecture I will attempt to introduce the subject and lay down what I hope is a simple yet solid basis which can be the starting point for further study.

Throughout I will be working with the following notation: comoving coordinates will be represented using lower-case letters (e.g., x) and proper distances with upper-case letters (e.g., D). The expansion of the Universe will be parameterised by $R(t)$ known as the scale factor of the Universe and t is the cosmological time. Critical densities will be represented by Ω and present-day values with a zero subscript, e.g., R_0.

The CMB observables

The frequency spectrum of the CMB radiation was measured to high accuracy in the early nineties by FIRAS (as part of the COBE mission, which also gave us the first full-sky map of the CMB), and it was found to be that of a black-body at a temperature T=2.72K over a large range of frequencies. This profile indicates thermal equilibrium and it is to date the best example of a black-body known in the Universe. This alone can tell us something about the early Universe. If we assume an adiabatic expansion (by which we mean the entropy change in any comoving region is zero), then $T \propto V^{(1-\gamma)}$, where γ is the ratio of specific heats and is equal to 4/3 for radiation. Volume goes as the cube of R and therefore $T \propto R^{3(1-\gamma)}$, which simply gives $T \propto 1/R$. So relating the present day temperature to the temperature at a redshift z and using the relation $R_0/R(z) = 1 + z$ gives

$$T_0 = \frac{T(z)}{1+z} \tag{1}$$

This allows us to estimate the temperature of the radiation at the time the CMB was created. Our best estimate for the last scattering surface (LSS) redshift z_{LS} is approximately 1100, which gives us a temperature of around 3000K at the time of last scattering. And since $\nu_0 = \nu/(1+z)$, we expect a black-body spectrum to remain so in an adiabatic expansion (recall the flux of a black-body $B_\nu = \frac{2h\nu^3c^2}{e^{h\nu/KT}-1}$).

However, the vast majority of information lies not in the frequency spectrum of the CMB, but in its temperature field. Although the observed average temperature is amazingly uniform across the sky, a good signal-to-noise experiment will reveal small fluctuations around this average. These fluctuations are small (1 part in 10,000!), and in 2003 the satellite experiment WMAP provided the first high resolution, high signal-to-noise, full-sky map of these fluctuations. Since we are interested in the deviation from the average temperature, we generally define a dimensionless quantity $\Theta(\hat{\mathbf{n}}) = \frac{T(\hat{\mathbf{n}})-\langle T\rangle}{\langle T\rangle}$, where $\hat{\mathbf{n}}$ is a direction in the sky, $\hat{\mathbf{n}} \equiv (\theta, \phi)$

We see these temperature fluctuations projected in a 2D spherical surface sky, and so it has become common in the literature to expand the temperature field using spherical harmonics. The spherical harmonics form a complete orthonormal set on the unit sphere and are defined as

$$Y_{lm} = \sqrt{\frac{2\ell+1}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}} P_\ell^m(cos\theta)e^{im\phi} \tag{2}$$

where the indices $\ell = 0, ..., \infty$ and $-\ell \leq m \leq \ell$ and P_ℓ^m are the Legendre polynomials. ℓ is called the multipole and represents a given angular scale in the sky α, given approximately by $\alpha = \pi/\ell$ (in degrees).

We can expand our temperature fluctuations field using these functions

$$\Theta(\hat{\mathbf{n}}) = \sum_{\ell=0}^{\ell=\infty}\sum_{m=-\ell}^{\ell} a_{lm}Y_{lm}(\hat{\mathbf{n}}) \tag{3}$$

where

$$a_{lm} = \int_{\theta=-\pi}^{\pi}\int_{\phi=0}^{2\pi}\Theta(\hat{\mathbf{n}})Y_{lm}^*(\hat{\mathbf{n}})d\Omega \tag{4}$$

and, analogously to what we do in Fourier space, we can define a *power spectrum* of these fluctuations, C_ℓ, as the variance of the harmonic coefficients

$$\langle a_{lm}a_{l'm'}^*\rangle = \delta_{\ell\ell'}\delta_{mm'}C_\ell \tag{5}$$

where the above average is taken over many ensembles and the delta functions arise from isotropy (we can sample from our own Universe). We only have one Universe, so we are intrinsically limited on the number of independent m-modes we can measure - there are only $(2\ell+1)$ of these for each multipole. We can write the following expression for the power spectrum:

$$C_\ell = \frac{1}{2\ell+1}\sum_{m=-\ell}^{\ell}\langle|a_{lm}|^2\rangle \tag{6}$$

This leads to an unavoidable error in our estimation of any given C_ℓ of $\Delta C_\ell = \sqrt{2/(2\ell+1)}$: how well we can estimate an average value from a sample depends on how many points we have on the sample. This is normally called the cosmic variance.

In real space, the power spectrum is related to the expectation value of the correlation of the temperature between two points in the sky:

$$\xi_{\Theta\Theta}(\theta) = \langle\Theta(\hat{\mathbf{n}})\Theta(\hat{\mathbf{n}}')\rangle = \frac{1}{4\pi}\sum_{\ell=0}^{\infty}(2\ell+1)C_\ell P_\ell\cos\theta,\ \hat{\mathbf{n}}.\hat{\mathbf{n}}' = \cos\theta \tag{7}$$

Cosmological models normally predict what the variance of the a_{lm} coefficients is over an ensemble, so they predict the power spectrum. Each Universe is then only one realisation of a given model.

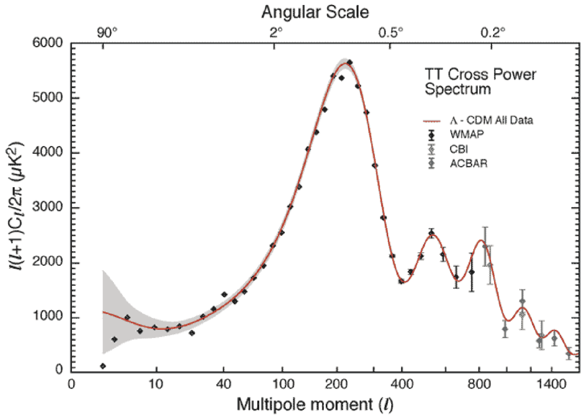

Figure 1: The CMB power spectrum as a function of angular scale. Red line is our best fit to the model, and the grey band represents the cosmic variance (see text).

Under the Inflation paradigm, the temperature fluctuations are Gaussian, which means that the harmonic coeffcients have Gaussian distributions with mean zero and variance given by C_ℓ. In this case, **all we need to characterise the statistics of our temperature fluctuations field is the power-spectrum - all higher-point statistics will be zero and contain no extra information**. The Gaussian hypothesis is now being questioned by detections of non-Gaussianity and deviations from isotropy in the WMAP first-year data. However, none of the detections are yet confirmed to be of cosmological origin and we continue to assume Gaussianity throughout this lecture.

The sum in equation (3) will generally start at $\ell = 2$ and go on to a given ℓ_{max} which is dictated by the resolution of the data. The reasons why we exclude the first two terms are as follows. The monopole ($\ell = 0$) term is simply the average temperature over the whole sky ($Y_{00} = 1/2\sqrt{\pi}$ which makes $\Theta(\hat{\mathbf{n}})_{\ell=0} = 1/4\pi \int\int \Theta(\hat{\mathbf{n}}) d\phi dcos\theta \equiv \langle\Theta(\hat{\mathbf{n}})\rangle$, where the integrals are done over the entire surface), and so from our definition of $\Theta(\hat{\mathbf{n}})$ it should average to zero. The monopole temperature term would be a valuable source of cosmological information in its own right, but its value can never be determined accurately because of cosmic variance - essentially we have no way of telling if the average temperature we measure locally is different from the average temperature of the Universe. The dipole term ($\ell = 1, \alpha \approx 180°$) is affected by our own motion across space - CMB photons coming towards us will appear blueshifted and those going away from us will appear redshifted. This creates an anisotropy at this scale which dominates over the intrinsic cosmological dipole signal and therefore we normally subtract the monopole/dipole from a CMB map or discard the first two values of the power spectrum prior to any analysis.

Our best estimate at what the power spectrum of the observed CMB fluctuations looks like can be seen in Figure 1. It is usually plotted as $\ell(\ell+1)C_\ell/2\pi$. This is related to the contribution towards the variance of the temperature fluctuations in a patch of sky of size $\propto 1/\ell$: $\langle\Theta^2\rangle = \xi_{\Theta\Theta}(0) = \frac{1}{4\pi}\sum_\ell(2\ell+1)C_\ell$ (since $P_\ell(1) = 1$). The contribution over a range of values of ℓ is given approximately given by $\int_\ell^\infty 2\ell' C l' d\ell' = \int_\ell^\infty 2\ell'^2 C'_\ell \frac{d\ell'}{\ell'}$ (for $\ell \gg 1$) and so $2\ell^2 C_\ell$ is proportional to the contribution to the variance per unit $\ln \ell$. This gives a flat plateau at large angular scales, and brings out a lot of the structure at smaller scales (see later).

Relating angular sizes with linear scales

Having had a look at what how we observe the CMB radiation today and having looked at some of the formalism we need to analyse it, we would like to start relating what we observe to what was happening at the time of last scattering. One of the first things we can do is to ask how we can relate angular scales in the sky to linear sizes at the time of last scattering.

We take the LSS as being a spherical surface at a distance z_{LS} from us. We will take the comoving distance to this surface as being r_{LS}. We want to relate a small angle in the sky θ to the linear comoving distance x

at last scattering, such that $\theta \approx x/r$ (for $\theta \ll 1$ and in flat space). The comoving distance-redshift relation is given by

$$R_0 r = \frac{c}{H_0} \int \frac{dz}{H(z)} \tag{8}$$

where $H(z)$ is the Hubble factor and is given by $H(z) = [\Omega(1+z)^2 + \Omega_v + \Omega_m(1+z)^3 + \Omega_r(1+z)^4]^{1/2}$. This integration can only be done numerically for most cases, but for the case of a matter dominated, flat Universe then the Hubble factor simplifies to $H(z) = (1+z)^{3/2}$ and we get

$$r_{LS} = \frac{c}{R_0 H_0} \int_0^{z_{LS}} (1+z)^{-3/2} dz = \frac{2c}{H_0 R_0} \left| -(1+z)^{-1/2} \right|_{z=0}^{z=z_{LS}} = \frac{2c}{H_0 R_0} (1 - (1+z_{LS})^{-1/2}) \tag{9}$$

For $z_{LS} \gg 1$ then r_{LS} is given simply by $\frac{2c}{H_0 R_0}$. Formally, by taking z_{LS} to infinity we are effectively calculating the present-day particle horizon length which is the maximum comoving distance light could have travelled since the Big Bang, d_H. So a small angle in the sky θ corresponds to a linear comoving distance at last scattering given by (in radians)

$$\theta = x \frac{R_0 H_0}{2c} \tag{10}$$

One particular comoving distance at the time of last scattering which we might be interested in is the particle horizon length, which is given by

$$d_H(z = z_{LS}) = \int_{z_{LS}}^{\infty} \frac{dz}{H(z)} = \frac{2c}{H_0 R_0} (1+z_{LS})^{-1/2} \tag{11}$$

which, from (10) and for $z_{LS} \approx 1100$, means that

$$\theta_H^{LS} = (1+z_{LS})^{-1/2} \approx 1.7^\circ \tag{12}$$

This tells us that scales larger than 1.7° in the sky were not in causal contact at the time of last scattering. However, the fact that we measure the same mean temperature across the entire sky suggests that all scales were once in causal contact - this led to the idea of Inflation, which suggests that the Universe went through a period of very fast expansion, which would have stretched a small, causally connected patch of the Universe into a region of size comparable to the size of the observable Universe today. Inflation theories usually make use of one or more repulsive potential fields to drive the inflationary period. Quantum fluctuations in these fields can then be stretched and propagated into gravitational potential perturbations, which will in turn leave their mark in the temperature and density fields (these are the seeds which will evolve under gravity to give rise to the large-scale structure of the Universe we see today). The whole subject of Inflation is extremely vast and complex and is beyond the point of this lecture. As far as we're concerned, Inflation is a mechanism which provides us with primordial spatial inhomogeneities in the gravitational field, $\delta\Phi$, and with uniformity across the whole sky.

Physical Mechanisms: the origin of the anisotropies

CMB anisotropies can be classified into primary or secondary anisotropies, according to whether they were created at last scattering or during the photon's path along the line of sight. Photons can be affected by a range of things after last-scattering e.g. re-ionization, passing through hot clusters gas, evolving potential wells, gravitational lensing, etc. While secondary anisotropies hold a good deal of information about the more recent Universe, they are not the subject of this lecture. Mostly, their effect on the temperature power spectrum lies at very small scales (very large ℓ) just beyond our current technical abilities. The exception is the Integrated Sachs-Wolf effect (related to time-evolving potential wells) whose effect shows up at very large scales (very small ℓ), and causes a slight rise in the power spectrum.

Our interest for today lies in the primary anisotropies. These, in turn, are created by two main mechanisms: gravitational and adiabatic.

$$\Theta = \Theta_{grav} + \Theta_{ad} \tag{13}$$

Perturbations in the gravitational potential $\delta\Phi$ left from Inflation can affect the radiation in two different ways. Firstly, through gravitational redshift, which in the weak field regime is given by

$$\frac{\delta\nu}{\nu} = \Theta \approx \frac{\delta\Phi}{c^2} \tag{14}$$

Secondly, by causing a time dilation at time scattering $\delta t/t = \delta\Phi/c^2$, which means we are looking at a younger universe at overdensities. In early times, $R \propto t^{2/3}$ and recalling that $T \propto 1/R$ we promptly get

$$\Theta \approx -\frac{2}{3}\frac{\delta\Phi}{c^2} \tag{15}$$

where we've again taken a weak-field approximation and assumed an adiabatic expansion. The added effect is simply

$$\Theta_{grav} \sim \frac{1}{3}\frac{\delta\Phi}{c^2} \tag{16}$$

which is commonly known as the Sachs-Wolf effect. These fluctuations happen at all scales, but dominate at large scales, where causal effects such as fluid dynamics (see next) don't come into account. For a spatial matter power-spectrum $P(k) \propto k^n$ (k is the wavenumber), the angular power-spectrum C_ℓ reduces to (for $n = 1$) $C_\ell \propto 1/\ell(\ell+1)$. This dependency gives rise to the flat part of the plot in Figure 1, which is usually called the Sachs-Wolf plateau.

We now turn to adiabatic perturbations. We've been slowly building up a picture of how the Universe was like at last scattering. Due to the high temperature, the Universe was fully ionized and consisted of a plasma mixture which, amongst others, contained photons and baryons. Thompson scattering meant that the photons were tightly coupled to the electrons which were in turn coupled to the baryons via Coulomb interactions. This coupling, together with radiation pressure acting as a restoring force, allows us to treat the primordial plasma as a perfect photon-baryon fluid to which normal fluid dynamics equations apply.

As mentioned before, the Universe also displayed small local potential wells into which matter falls. These potential perturbations, $\delta\Phi$ can be related to matter density perturbations by Poisson's equation $\nabla^2\Phi = 4\pi G\rho_m$. For an adiabatic expansion, the matter density perturbations are related to the radiation density and temperature perturbations by

$$\frac{1}{3}\frac{\delta\rho_m}{\rho_m} = \frac{1}{4}\frac{\delta\rho_\gamma}{\rho_\gamma} = \Theta_{ad} \tag{17}$$

remembering that $\rho_m \propto R^{-3}$ and $\rho_\gamma \propto R^{-4}$.

What we would like to do is study the dynamics of these temperature perturbations within this system. Let us take a simple model, in which we ignore gravity and the effect of the mass/intertia of the baryons (we're essentially taking a photon fluid), and see what happens to these temperature fluctuations under the influence of radiation pressure over time.

Remember now we are talking about *spatial* density and temperature perturbations, we're not working in the spherical sky any more. We'll be working in Fourier space: since the perturbations are small and evolve linearly we expect each k-mode to be independent.

The first thing to appreciate is that number of photons is conserved. We can write down a continuity equation for photon number density, n_γ, as

$$\dot{n}_\gamma + \nabla.(n_\gamma \mathbf{v}_\gamma) = 0 \tag{18}$$

where the derivative is with respect to conformal time $\eta \equiv cdt/R(t)$, which scales out the expansion, and $\mathbf{v}_\gamma$ is the photon fluid velocity. Taking into account the Universe's expansion, what is actually conserved is n_γ/R^3, and so $\dot{n}_\gamma + 3n_\gamma\frac{\dot{R}}{R} + \nabla.(n_\gamma\mathbf{v}_\gamma) = 0$

For linear perturbations $\delta n_\gamma = n_\gamma - \bar{n}_\gamma$ this reduces to

$$\dot{\left(\frac{\delta n_\gamma}{n_\gamma}\right)} = -\nabla.\mathbf{v}_\gamma \tag{19}$$

We can relate this to temperature fluctuations by $n_\gamma \propto T^3$ to give $3\Theta = \delta n_\gamma/n_\gamma$. This reduces our continuity equation to $\dot{\Theta} = -(1/3)\nabla.\mathbf{v}_\gamma$ or, in Fourier space, to

$$\dot{\Theta} = -\frac{1}{3}i\mathbf{k}.\mathbf{v}_\gamma \tag{20}$$

We now consider the momentum of the radiation. Momentum is given by $\mathbf{q} = (\rho_\gamma + p_\gamma)\mathbf{v}_\gamma$ where $(\rho_\gamma + p_\gamma)$ is the effective mass and, for radiation, $p_\gamma = (1/3)\rho_\gamma$. Ignoring gravitational effects and viscosities, the only force is given by the pressure gradient $\nabla p_\gamma = (1/3)\nabla\rho_\gamma$. We can then write $\dot{\mathbf{q}} = \mathbf{F}$ as

$$\frac{4}{3}\rho_\gamma\dot{\mathbf{v}}_\gamma = \frac{1}{3}\nabla\rho_\gamma \tag{21}$$

and so $\dot{\mathbf{v}}_\gamma = \nabla\Theta$ or, in Fourier space,

$$\dot{\mathbf{v}}_\gamma = ik\Theta \tag{22}$$

We now consider only the velocity component along the direction $\hat{\mathbf{k}}$, as this is the only one with a gravitational source and we write our final continuity and Euler equations as

$$\dot{\Theta} = -\frac{1}{3}kv_\gamma \text{ (Continuity)} \tag{23}$$

$$\dot{v}_\gamma = k\Theta \text{ (Euler)} \tag{24}$$

These can quickly be combined to give

$$\ddot{\Theta} + \frac{1}{3}k^2\Theta = 0 \tag{25}$$

which is a simple harmonic oscillator equation. The 1/3 factor is generally the adiabatic sound speed which is defined as $c_s^2 \equiv \dot{p}_\gamma/\dot{\rho}_\gamma$ which in this case is equal to 1/3. The general solution for equation (25) is given by

$$\Theta(\eta) = \Theta(0)\cos(kc_s\eta) + \frac{\dot{\Theta}(0)}{kc_s}\sin(kc_s\eta) \tag{26}$$

By assuming negligible initial velocities and by defining a sound horizon as $s \equiv \int c_s d\eta$, we simplify our solution to $\Theta(\eta) = \Theta(0)\cos(ks)$.

Let us take some time to revise what we have done. We are trying to analyse the dynamical behaviour of a photon-baryon fluid, and study how temperature fluctuations behave in this system. We took some very constraining assumptions (such as ignoring gravity and the baryons!) and worked on a system whose only force was given by radiation pressure gradients. What we found is that this **pressure acts as a restoring force to initial perturbations and we are left with oscillations which propagate at the speed of sound**. This is an important results, which holds even when we take into account other effects to make our system a realistic one. This behaviour continues until we hit the temperature of recombination, at which time matter and radiation de-couple and any temperature fluctuations are essentially frozen into the photons' temperature, which we measure (nearly unchanged!) today.

I emphasise that these oscillations are happening at all scales, and we are interested in those which at the time of recombination happen to be at their maxima or minima (remember, we're interested in the temperature squared). If this happens at a conformal time η_{rec} (corresponding to a sound horizon s_{rec}), then modes will be frozen with an amplitude given by

$$\Theta(\eta_{rec}) = \Theta(0)\cos(ks_{rec}) \tag{27}$$

and those caught at their extrema will have $k_n s_{rec} = n\pi$. We can therefore find a fundamental scale of oscillation by taking $n = 1$

$$k_F = k_1 = \frac{\pi}{s_{rec}} \tag{28}$$

This is our largest oscillating mode, and of course, all of the corresponding overtones will be caught at their extrema too. These will correspond to higher values of k_n and are simply oscillations which have had time to go another complete half-oscillation: k_1 corresponds to the oscillation which has had time to compress fully once, $k_2 = 2k_1$ to the oscillation which has had time to compress and then decompress fully, and so on.

We see that the maximum scale at which these fluctuations will happen (related to $\frac{1}{k_F}$) is related to the sound horizon at the time of recombination, which was close to the particle horizon. This means that scales larger than this won't be affected by acoustic oscillations, and we wouldn't expect otherwise given that acoustic oscillations can only happen in regions which are causally connected.

However, there is a caveat to this toy model. These oscillations also set up velocities in the fluid, which will in turn produce Doppler shifts in the frequencies of the photons. Velocity oscillations are precisely $\pi/2$ out of phase with acoustic oscillations (fluid stationary at maximum compression/extension), which in this case cancels the temperature oscillations in the radiation completely and gives a flat Θ.

A full treatment should take into account gravity, mass and inertia of the baryons, the evolution of the photon/baryon ratio, viscosity, diffusion and so on. A full solution looks more like (Hu, 1995):

$$< \frac{\Delta T}{T}^2(\eta) >= \left[\frac{1}{3}(1+3R)\Phi\cos(kc_s\eta) - R\Phi\right]^2 + \left[\frac{1}{3}(1+R)(1+R)^{-1/2}\Phi\sin(kc_s\eta)\right]^2 \tag{29}$$

where R is fluid momentum density and $R \equiv \frac{3\rho_b}{4\rho_\gamma} \approx \frac{450}{1+z}\frac{\Omega_b h^2}{0.015}$. The first term represents the acoustic oscillations which are equivalent to what we derived with our toy model and the second term represents the Doppler oscillations, which in this case are much smaller than the acoustic oscillations, but still $\pi/2$ out of phase (notice the sine instead of the cosine).

However, we don't need a full treatment to understand what the effect of gravity and the introduction of baryons does to this system, at least qualitatively. Gravity's main effect is to introduce another force into the system, $\mathbf{F}_{grav} = -m\nabla\Phi$, which adds a term to our Euler equations. This in effect creates a potential well and changes the range of the temperature oscillations, effectively their amplitude. If we now introduce baryons to the system, then we are introducing mass into the system. As in a classical system consisting of a spring (restoring force) and a mass attached to the end of ths spring (the photon-baryon fluid), increasing the mass will cause the spring to *fall* further, but it won't change the maximum rebound height. Recall that our peaks in the temperature fluctuations alternate between compression and expansion of the plasma, so introducing matter into the system changes only every other peak - those corresponding to compression of the fluid (matter is falling further into the potential well). Hence we expect even numbered peaks to be suppressed in relation to odd numbered peaks.

We also expect curvature to affect the observed angular temperature anisotropies, as it affects the path the CMB photons will have taken to get to us. It is essentially a problem of geometry: in a closed Universe, a given angle subtended in the sky will correspond to a smaller linear distance at last scattering than in a flat Universe, and so curvature shifts the peaks along the multipole axis. The detection of the first acoustic peak at an $\ell \approx 200$ provided a good constraint on the flatness of our Universe.

At smaller scales, however, we see that these oscillations are clearly damped. This comes from the fact that the last scattering surface has a finite thickness, and therefore recombination and last scattering don't happen at the same time. This damps out small scale fluctuations (where the scale is related to the thickness of the scattering surface), as photons still have to random walk out of this shell before they are essentially free, smoothing out the fluctuations.

Hopefully, it is clear now that the temperature fluctuations we see in the sky hold a vast amount of information about our Universe's content and evolution. And in fact, if the CMB is Gaussian, all of this information will be imprinted in the angular power spectrum C_ℓ. The extraction of information about cosmological parameters from the power spectrum is a complicated process, and it is worth mentioning that there are a number of degeneracies between these parameters which can only be lifted by combining CMB data with other data sets (such as supernova experiments or galaxy surveys). Nevertheless, CMB analysis is now precision Cosmology, which promises to continue to help us to construct a better image of our Universe.

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