



# Flexed universe

*Massimo Viola*

*DEX Meeting 13/01/2012*

# What is gravitational flexion ?

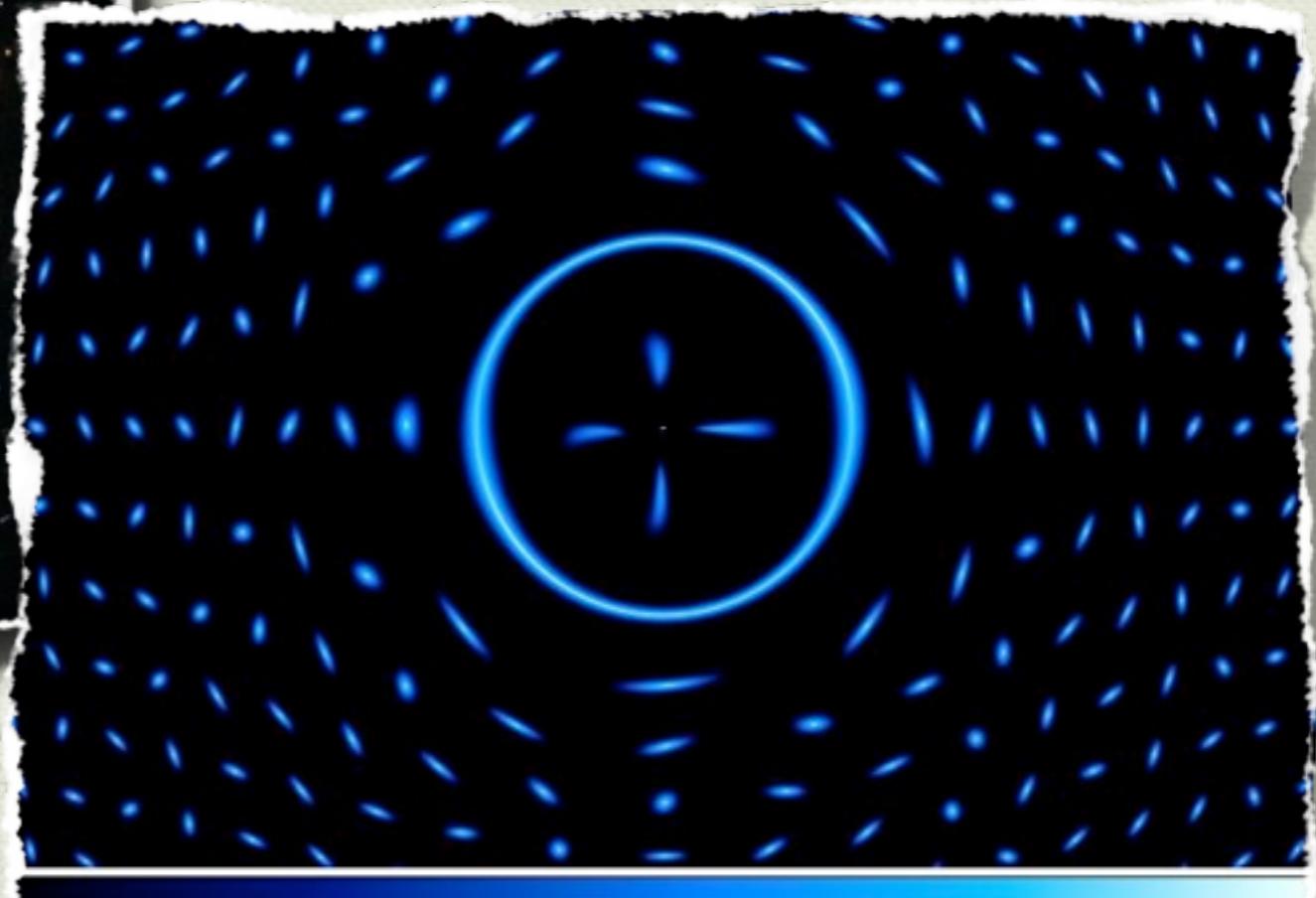


Abell 370 (HST)

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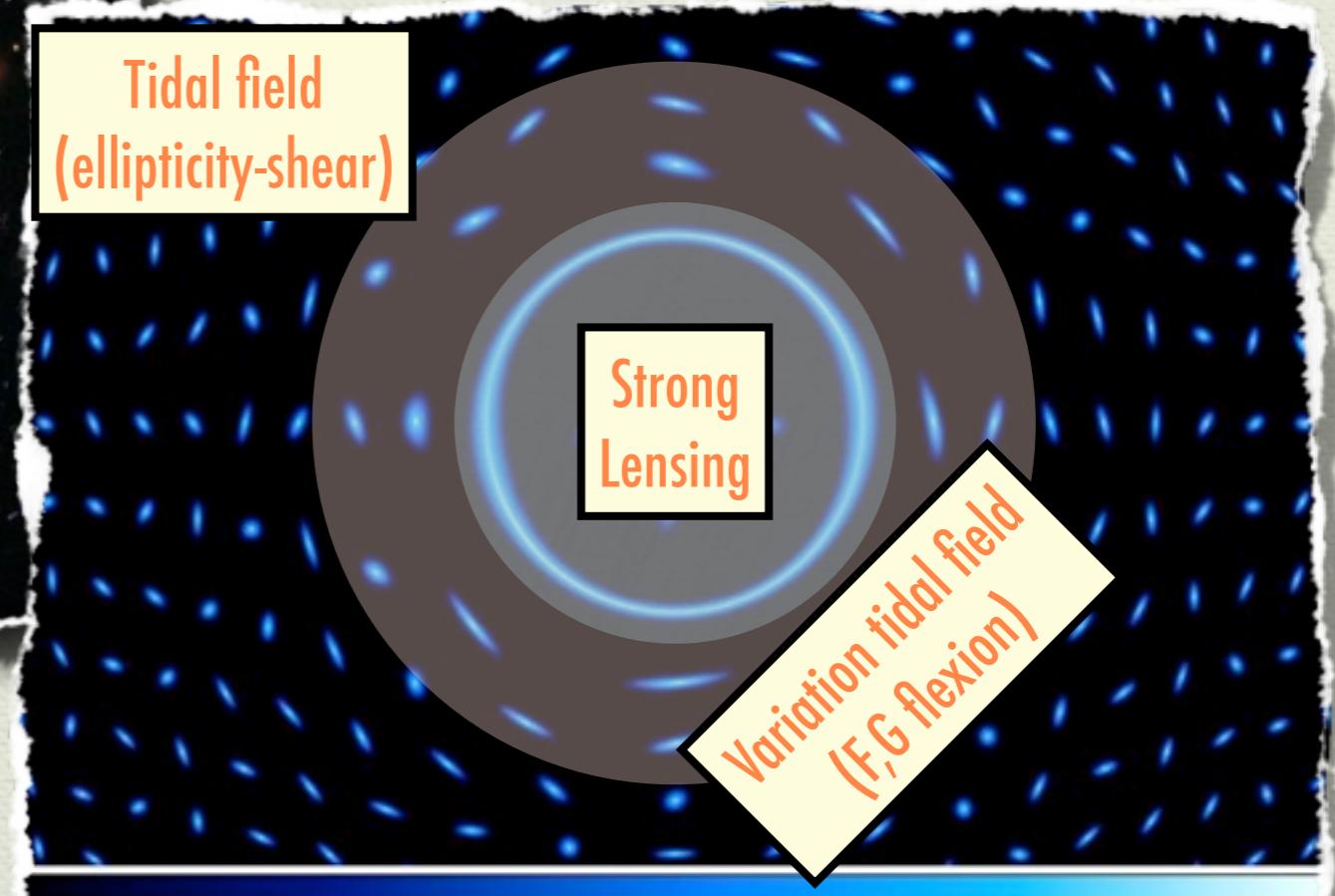


Abell 370 (HST)

Tidal field  
(ellipticity-shear)

Strong  
Lensing

Variation tidal field  
(F,G flexion)



# What is gravitational flexion ?

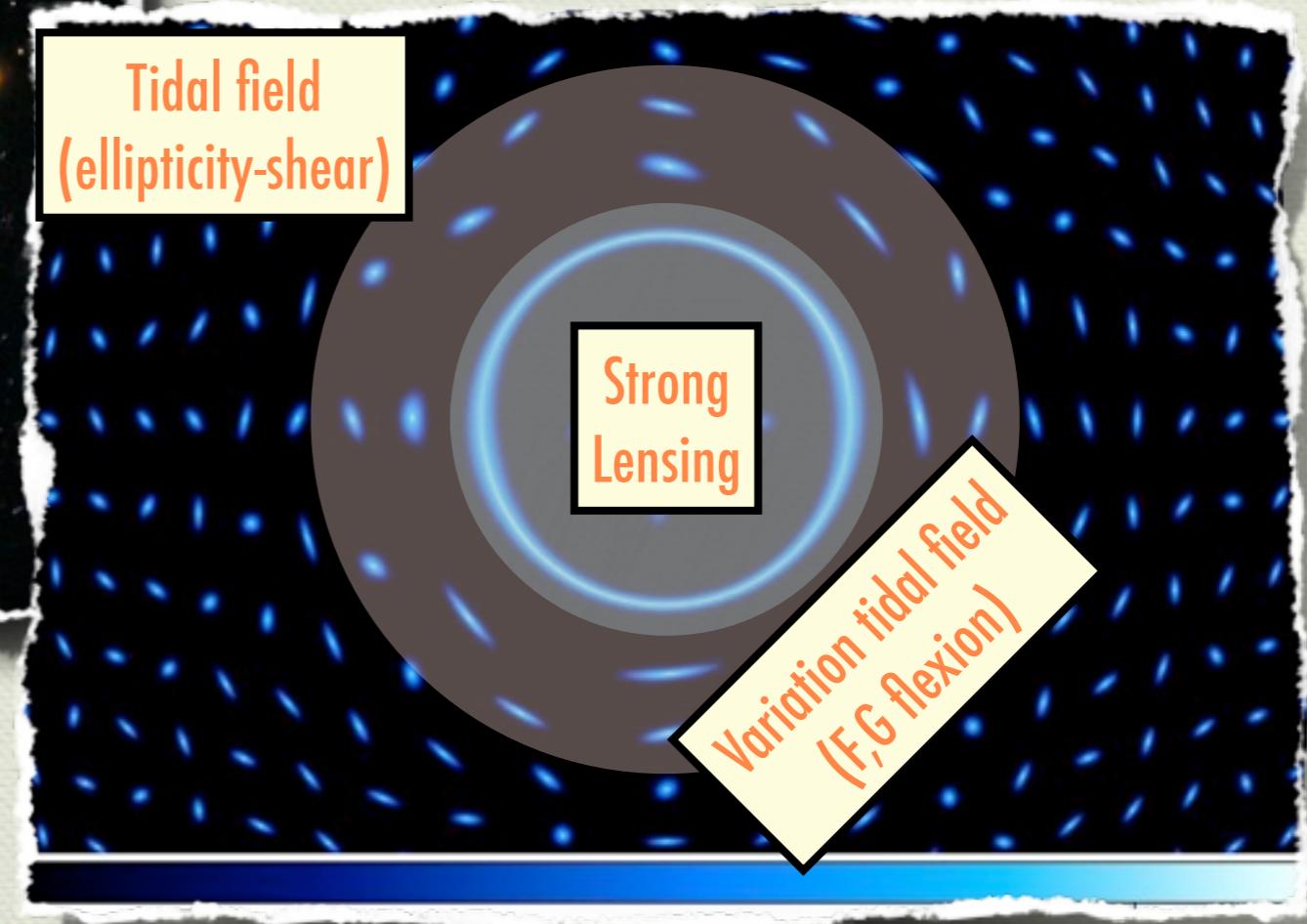
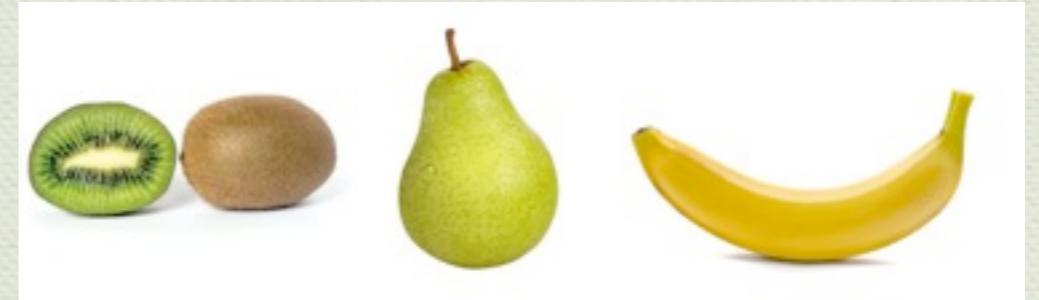


Abell 370 (HST)

Tidal field  
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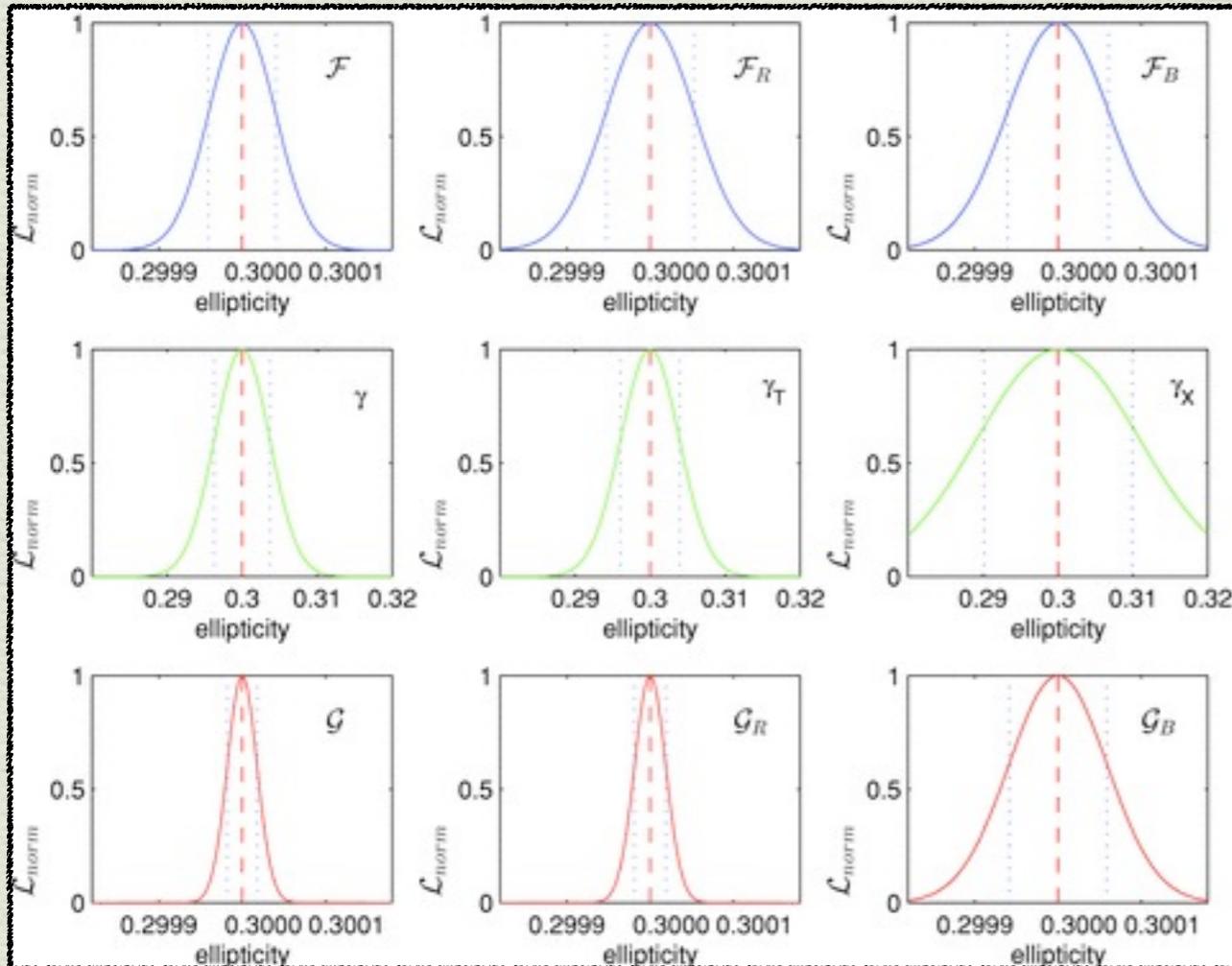
Strong  
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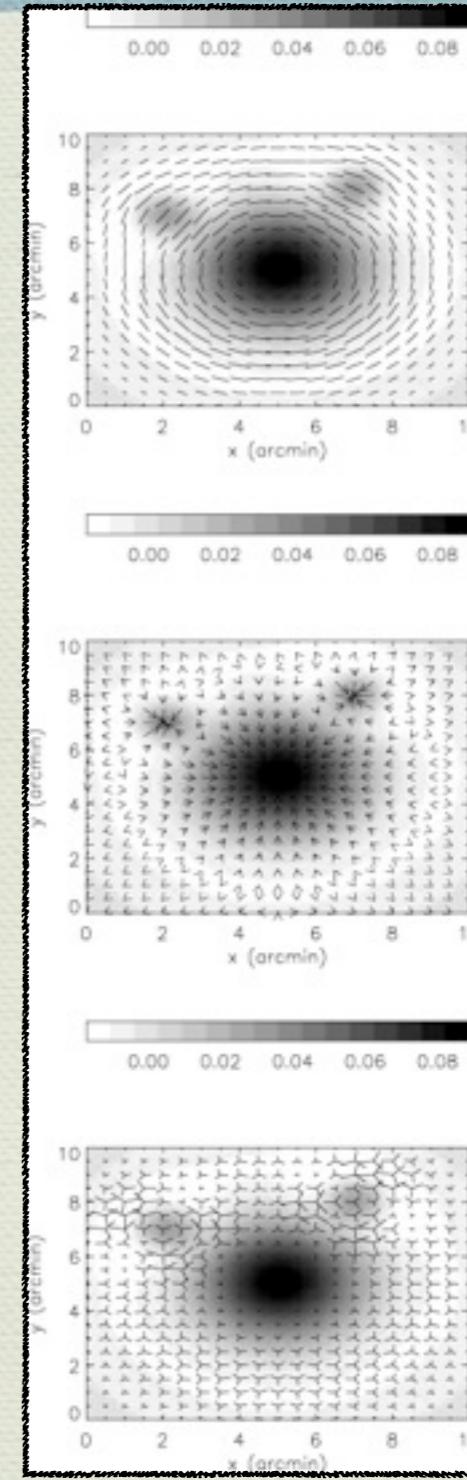
# Why is flexion interesting ?

20000 deg<sup>2</sup> & 35 gal/arcmin<sup>2</sup>

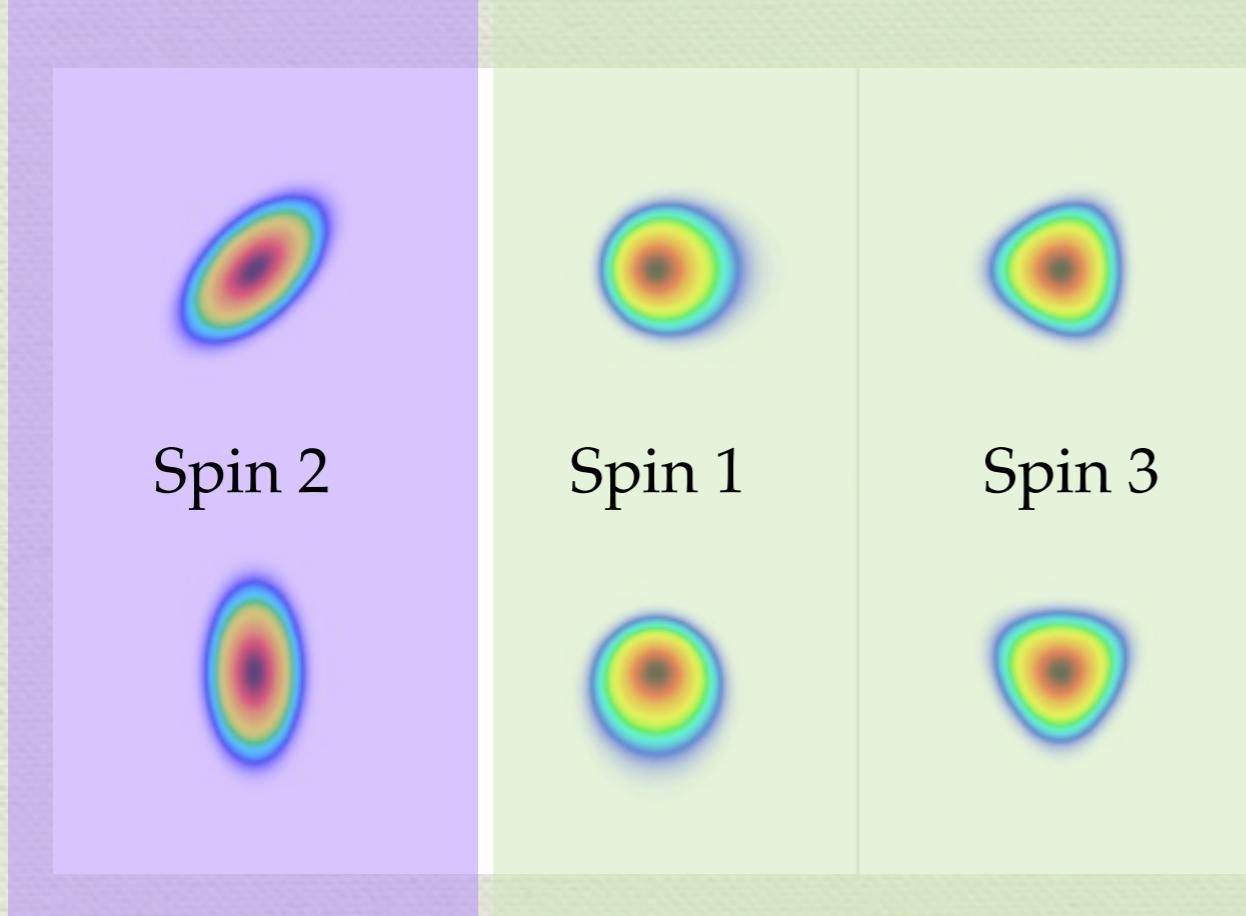


Hawken & Bridle 2009

Bacon et al. 2006



# The game is about measuring shapes...



Deformation = Intrinsic + induced by lensing

Intrinsic deformation is expected to vanish **on average**.

$$Q_{ij\dots k} = \int I(\vec{\theta}) \theta_i \theta_j \dots \theta_k d^2\theta$$

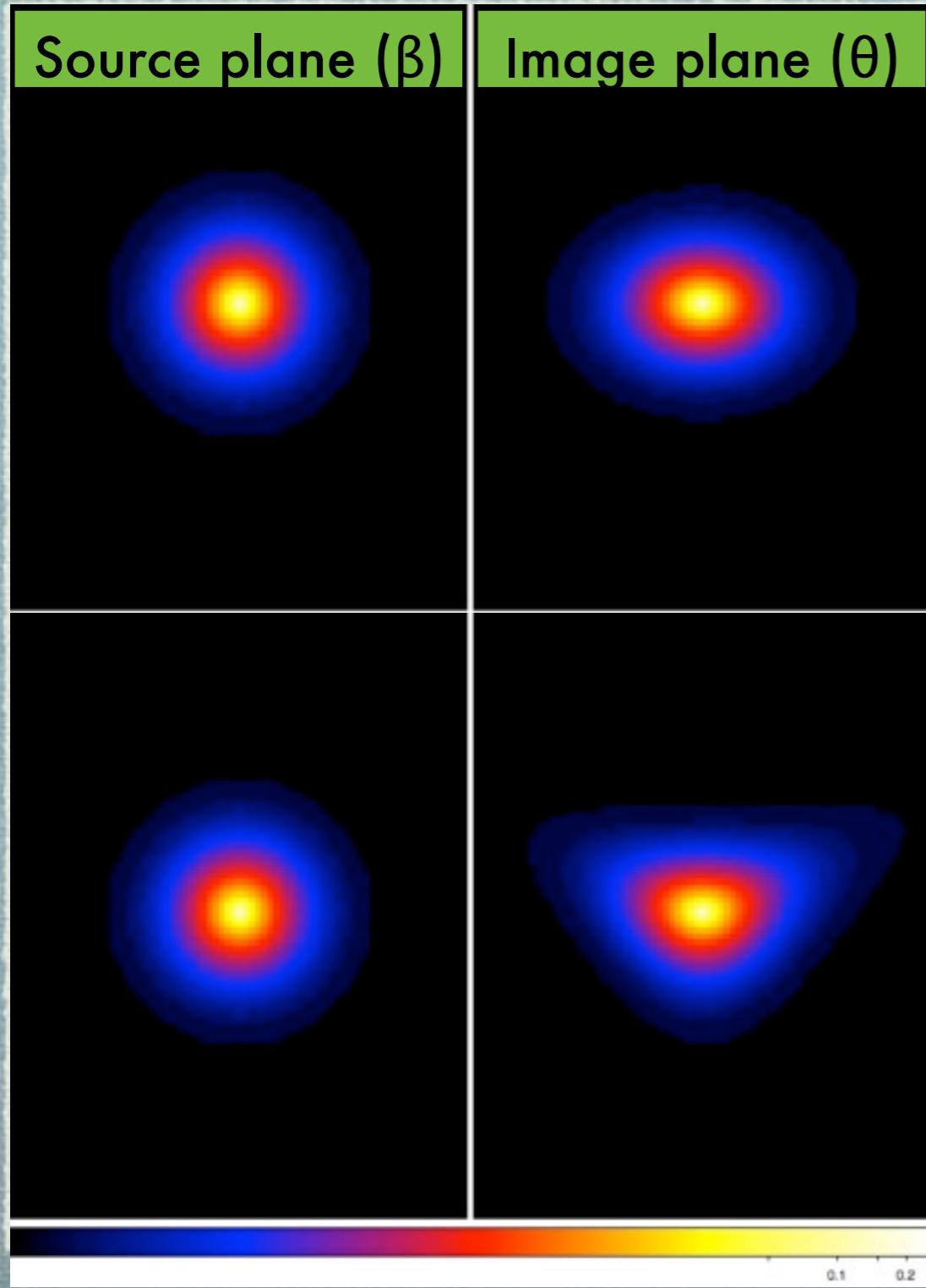
$$\chi = \frac{(Q_{11} - Q_{22}) + 2iQ_{12}}{Q_{11} + Q_{22}}$$

$$\zeta \equiv \frac{(Q_{111} + Q_{122}) + i(Q_{112} + Q_{221})}{\xi}$$

$$\delta \equiv \frac{(Q_{111} - 3Q_{122}) + i(3Q_{112} - Q_{221})}{\xi}$$

$$\xi \equiv Q_{1111} + 2Q_{1122} + Q_{2222}$$

# ...and doing some math!



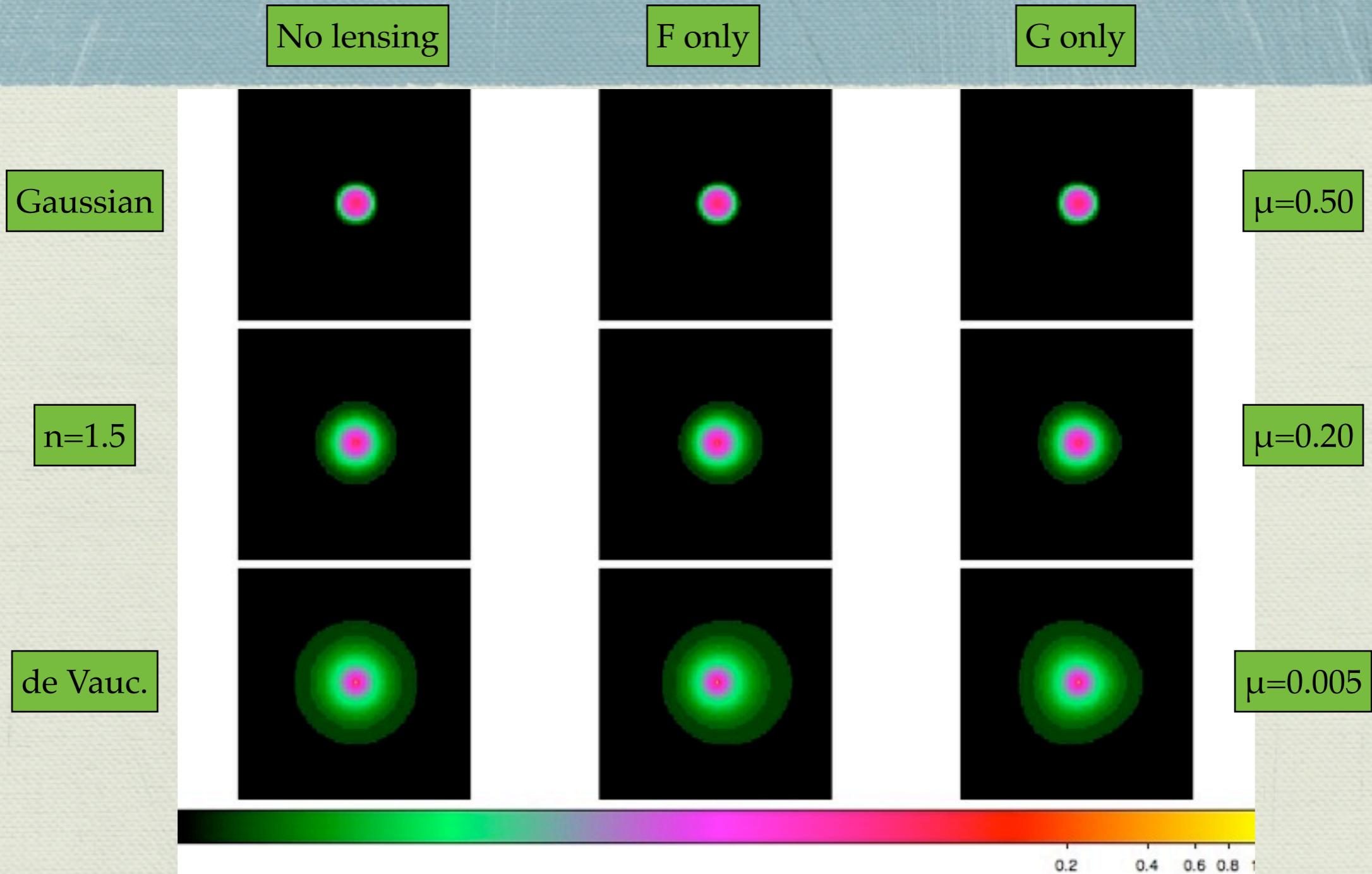
$$\chi^s = \frac{\chi - 2g + g^2\chi^*}{1 + |g|^2 - 2\Re(g\chi^*)}$$

$$\zeta^s \simeq \zeta - 2g\zeta^* - g^*\delta - \frac{9}{4}F + 3F^*g\dots + \mu \left( gG^*\chi + 3F + Gg^* + \dots \right)$$
$$\delta^s \simeq \delta - 3g\zeta - \frac{3}{4}G + 6Fg + \dots + 3\mu \left( -3Fg + \dots \right)$$

	Depends on scale	Depends on morphology
Shear	✗	✗
Flexion	✓	✓

# Flexion depends on morphology

Viola et al. 2011



$$\mu \equiv \frac{(Q_{11} + Q_{22})^2}{Q_{1111} + 2Q_{1122} + Q_{2222}}$$

# Simplest statistic: average

$$\langle \zeta \rangle \simeq \left( \frac{9}{4} - 3\langle \mu \rangle - \frac{5}{4}\langle \mu |\chi|^2 \rangle \right) F + \left( \frac{9}{4} - \frac{5}{4}\langle \mu |\chi|^2 \rangle - \langle \mu \rangle \right) G g^* + \left( \frac{15}{2} - 3\langle \mu |\chi|^2 \rangle - 2\langle \mu \rangle \right) F^* g - 4\langle \mu \chi \rangle F^* - \frac{1}{2}\langle \mu \chi^* \rangle G$$
$$\langle \delta \rangle \simeq \frac{3}{4}(1 - \langle \mu |\chi|^2 \rangle)G + \left( \frac{33}{4} - \frac{3}{4}\langle \mu |\chi|^2 \rangle \right) gF - \frac{9}{2}\langle \chi \mu \rangle F$$

General

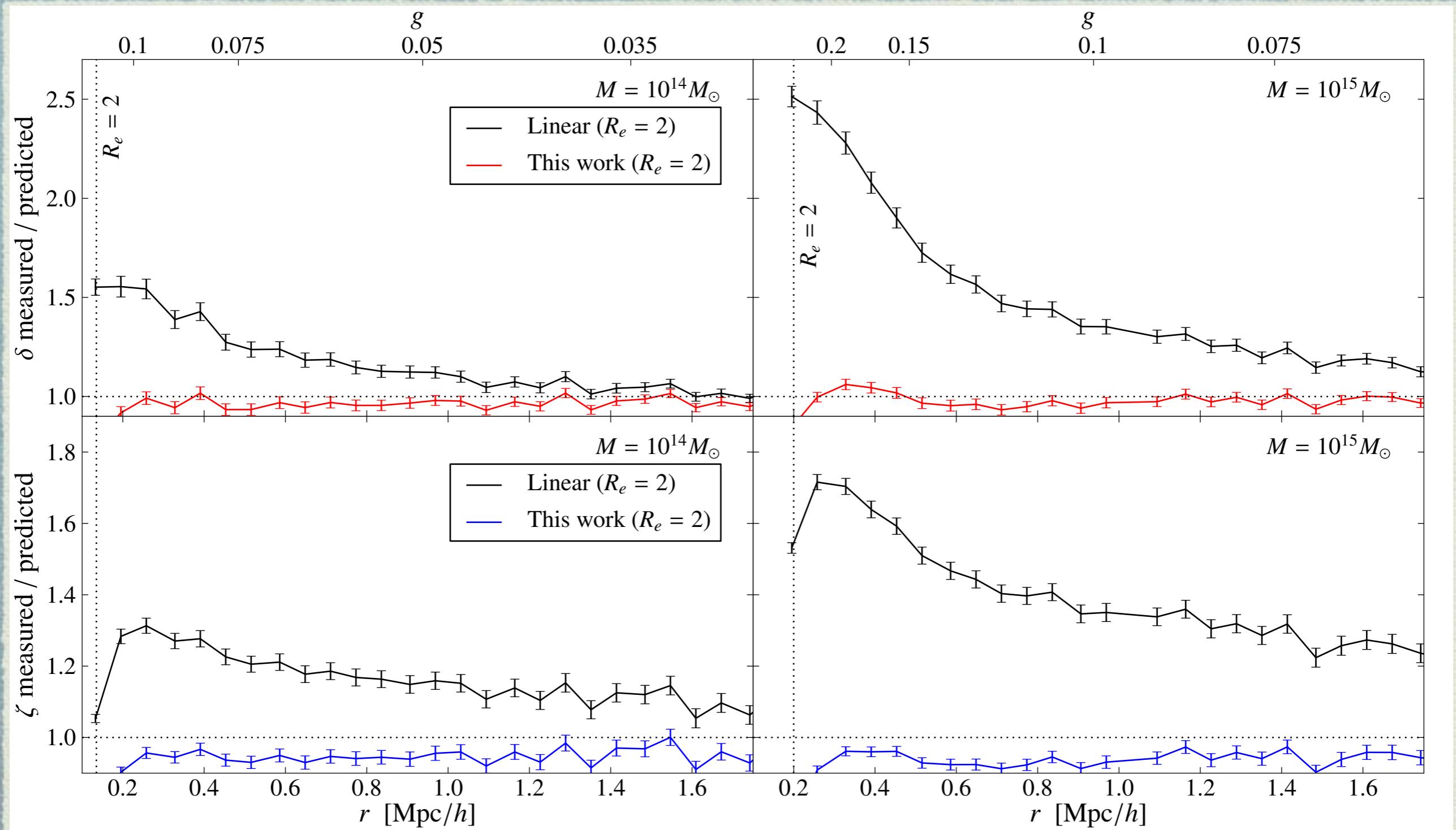
$$\langle \zeta \rangle \simeq \left( \frac{9}{4} - 3\langle \mu \rangle \right) F$$
$$\langle \delta \rangle \simeq \frac{3}{4}G$$

Okura et al. 2007,2008  
Leonard et al. 2007,2010

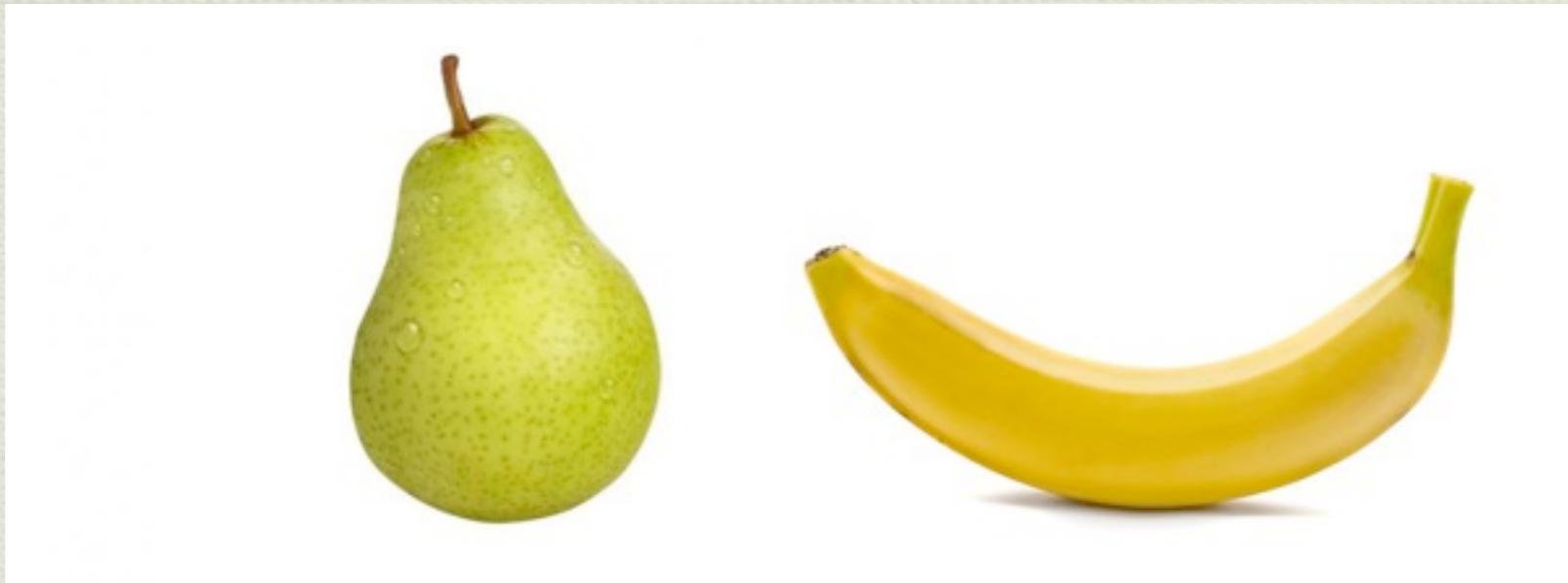
Viola et al. 2011

# What do we measure ?

Viola et al. 2011

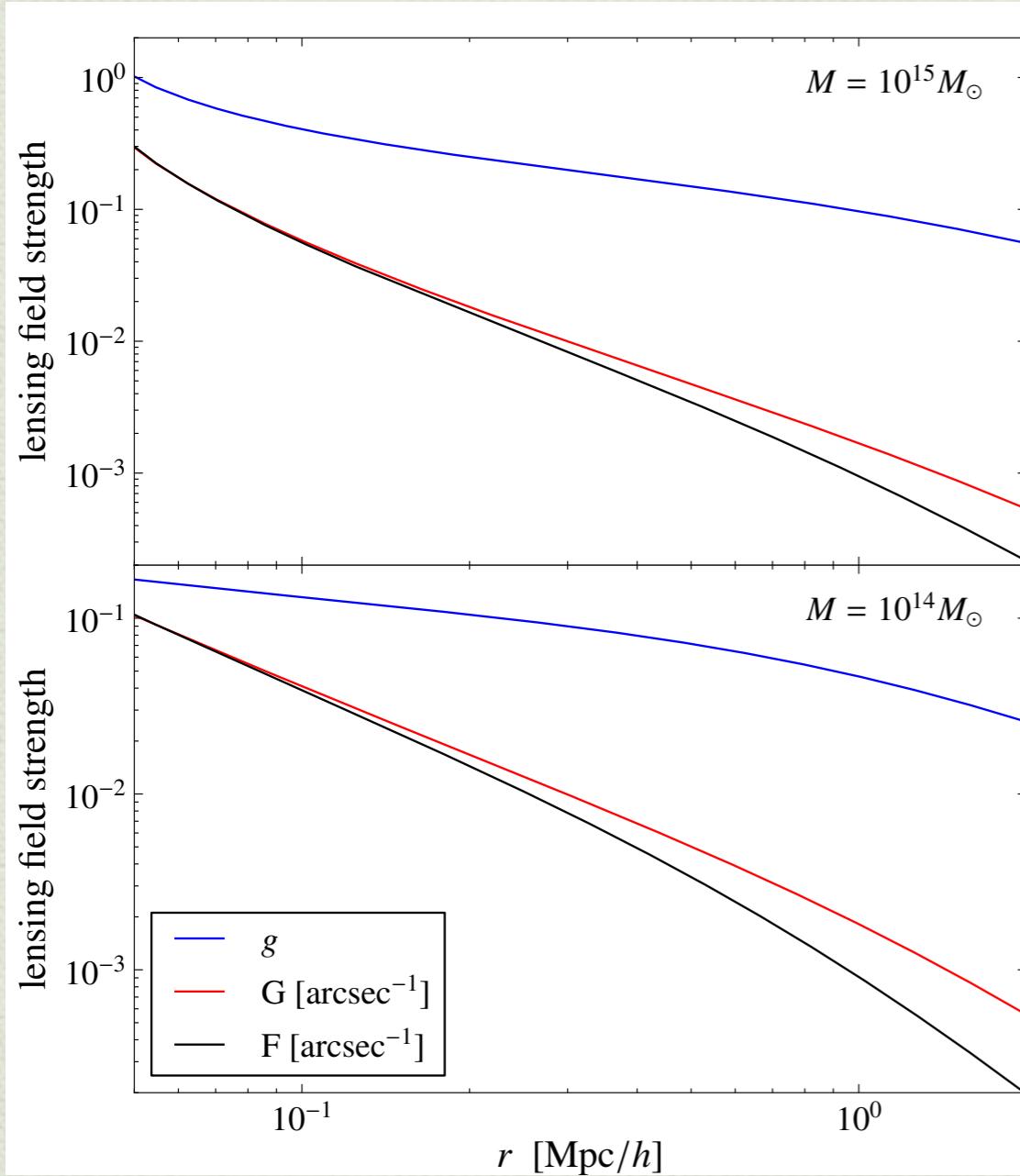


# Spin-1 or Spin-3 ?

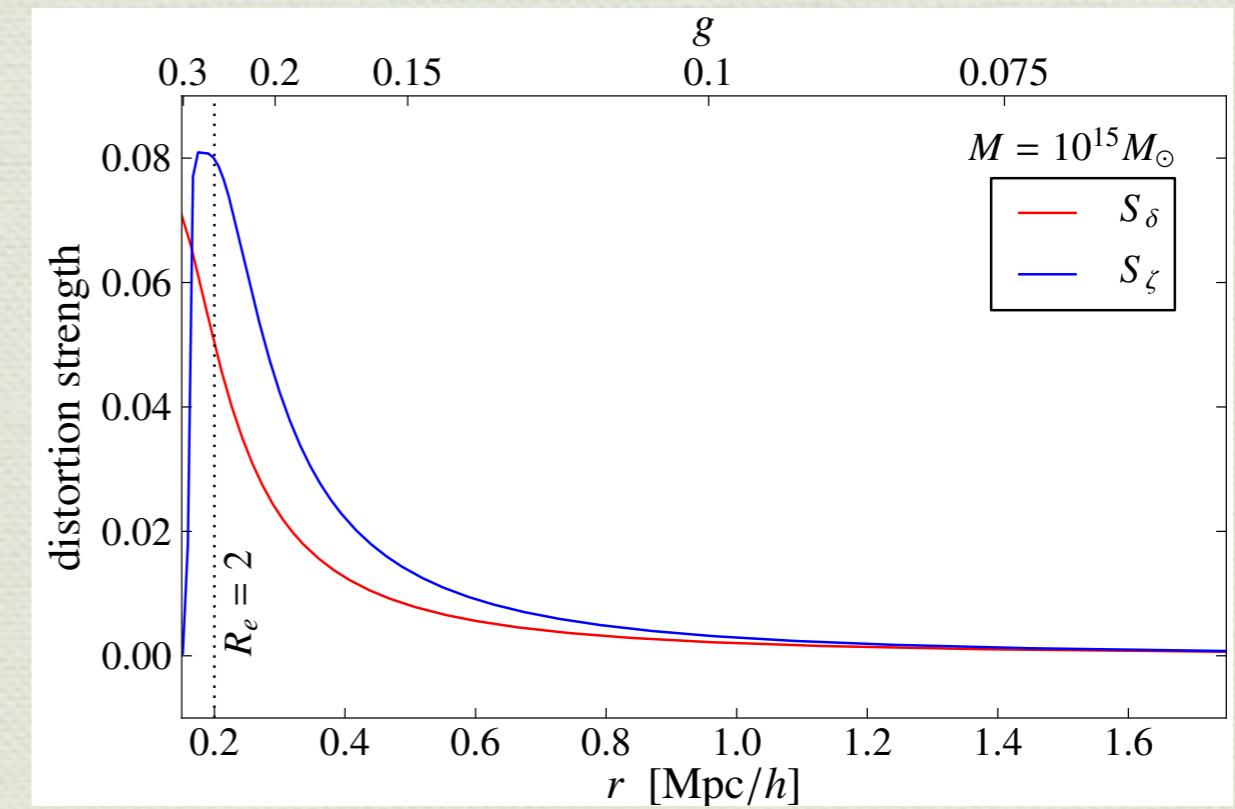
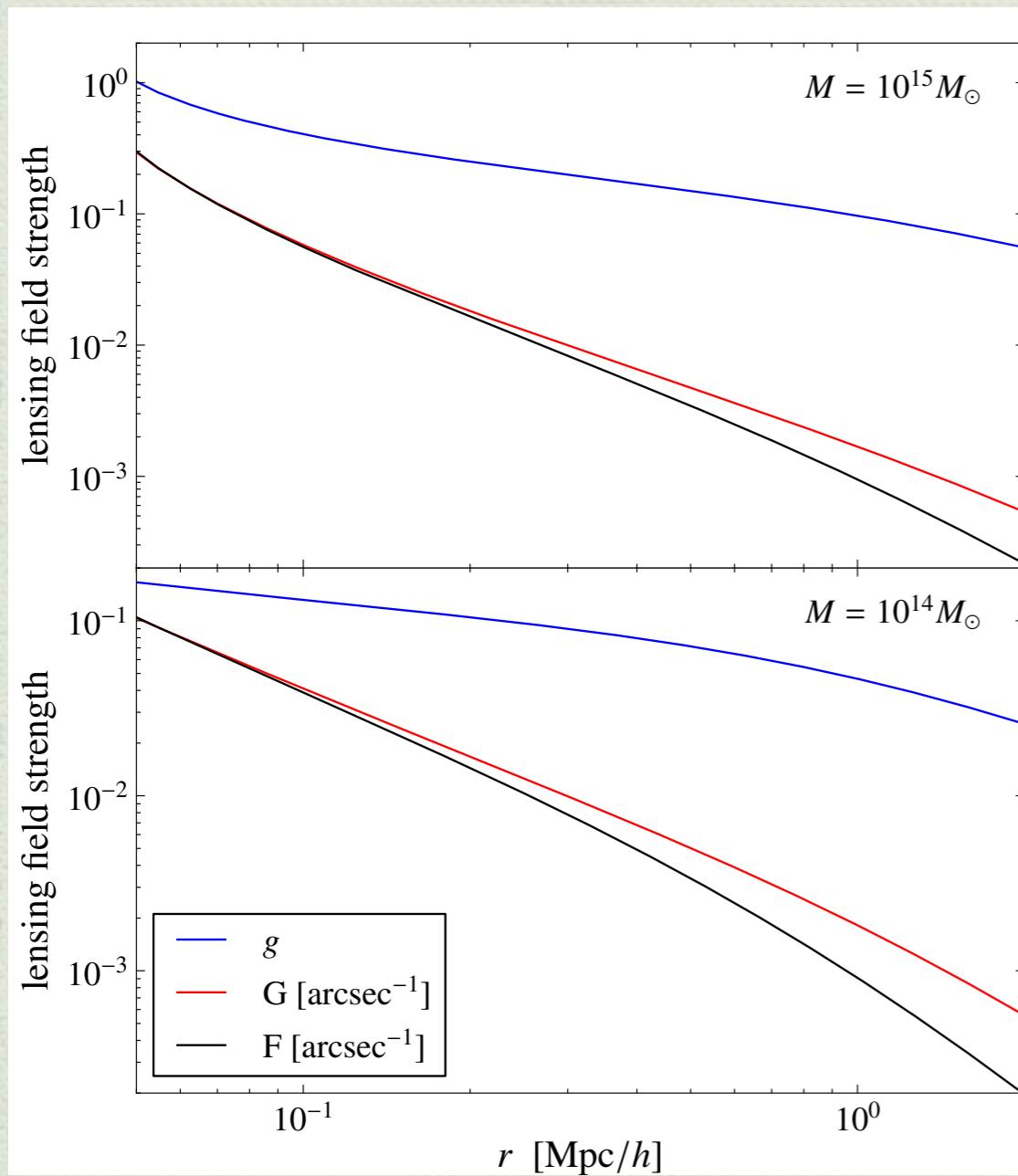


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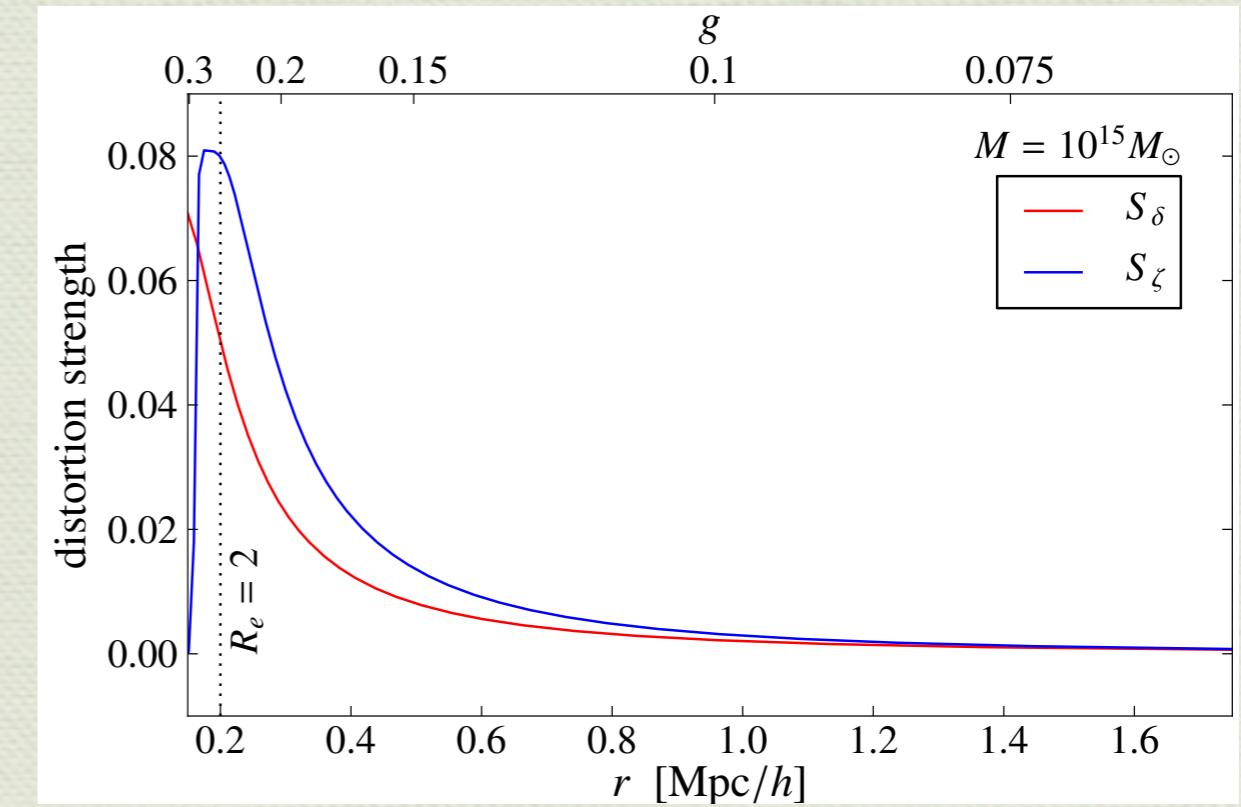
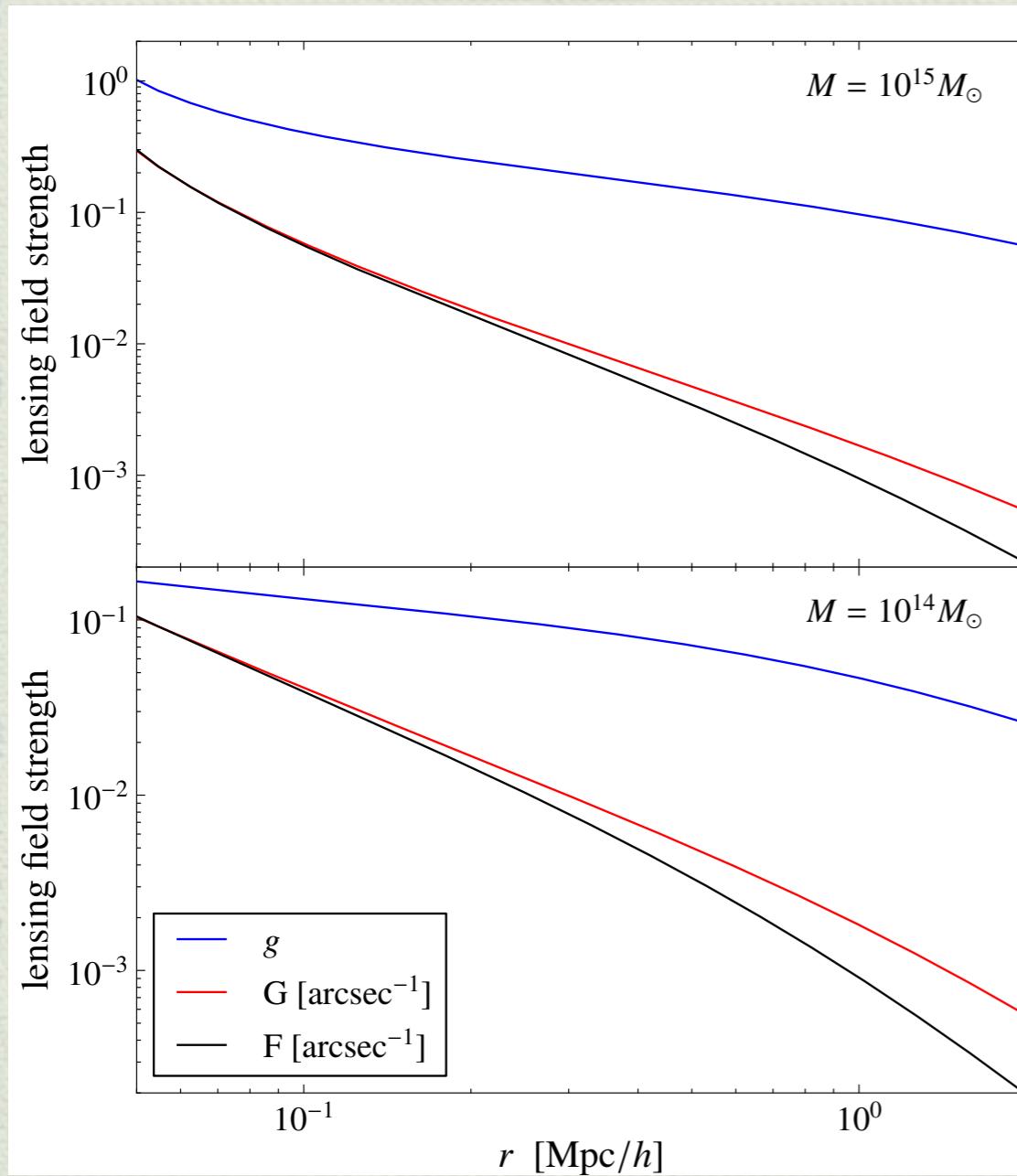
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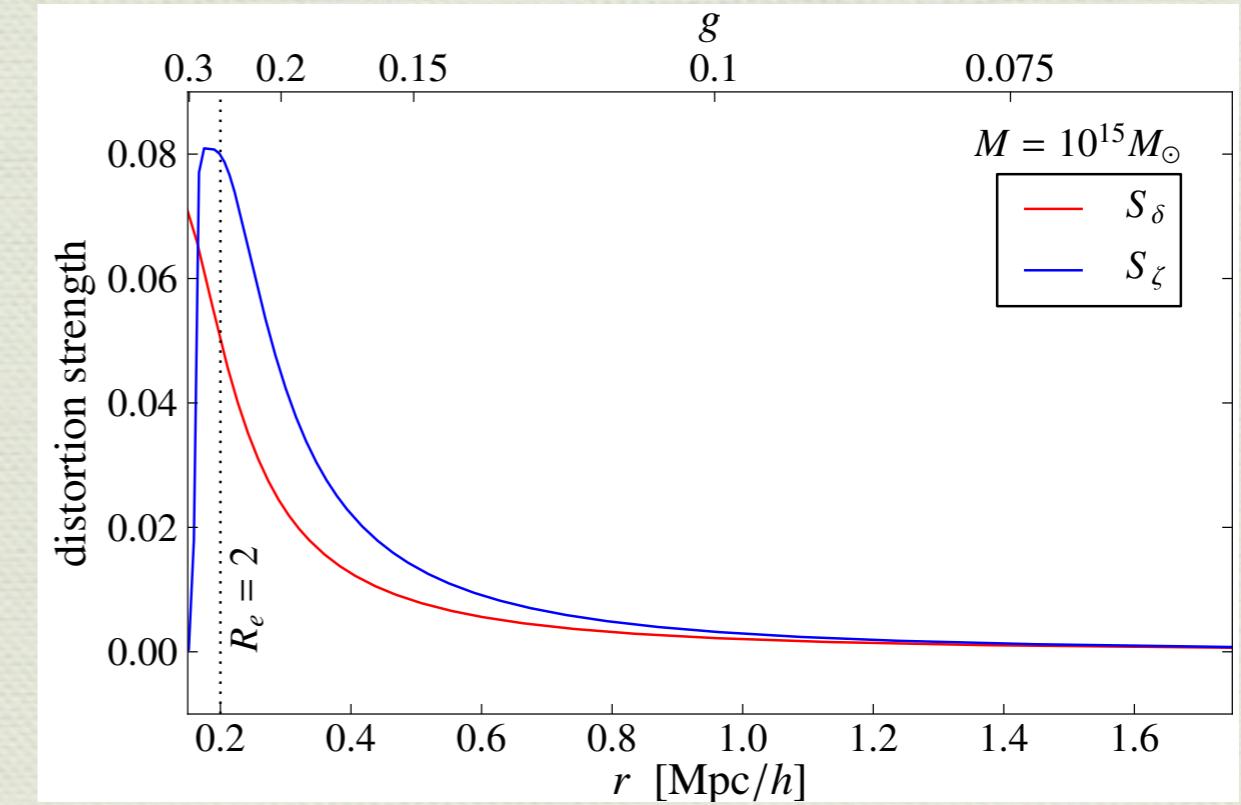
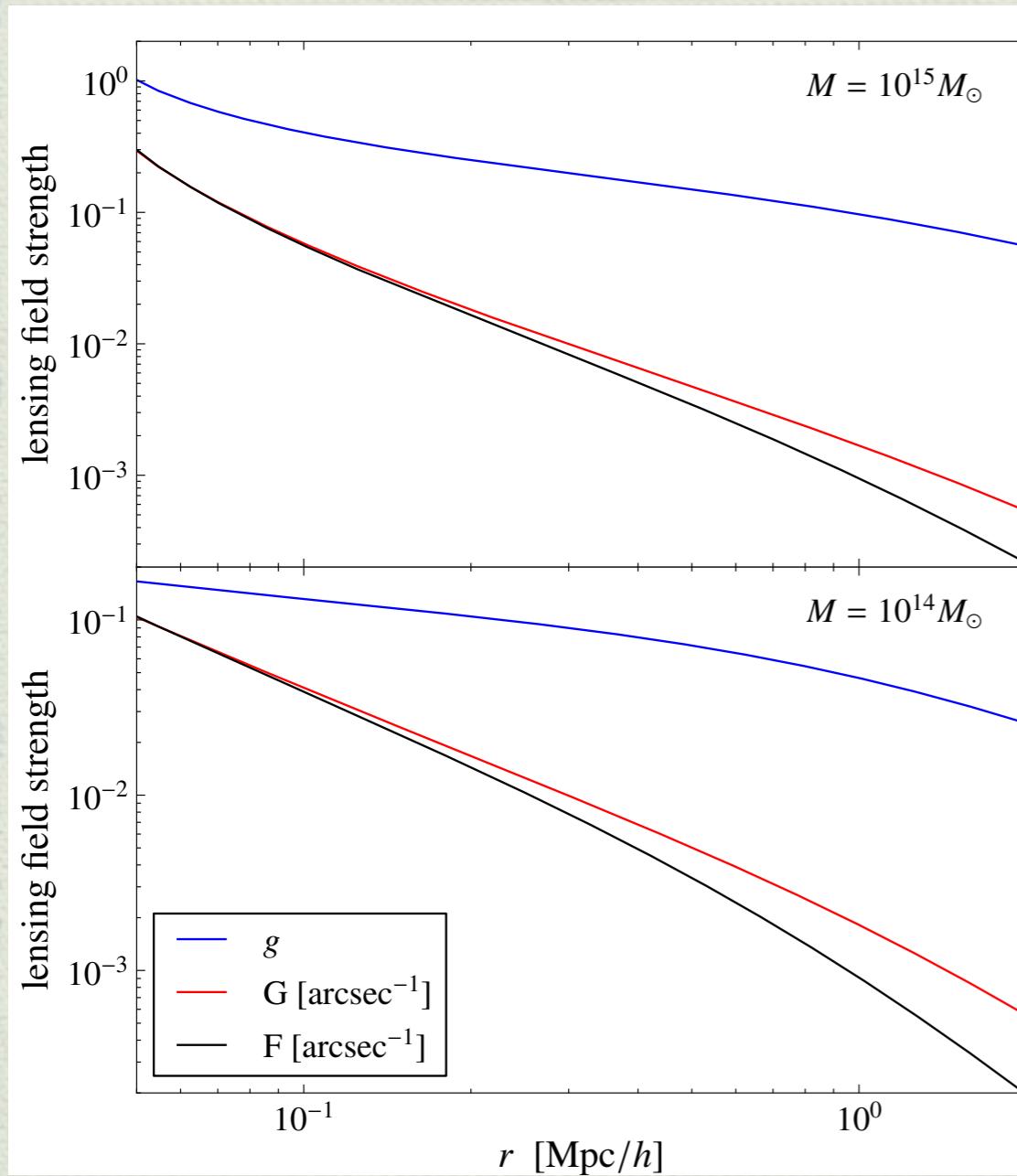
# Spin-1 or Spin-3 ?



$$\zeta \equiv \frac{(Q_{111} + Q_{122}) + i(Q_{112} + Q_{222})}{\xi}$$

$$\delta \equiv \frac{(Q_{111} - 3Q_{122}) + i(3Q_{112} - Q_{222})}{\xi}$$

# Spin-1 or Spin-3 ?



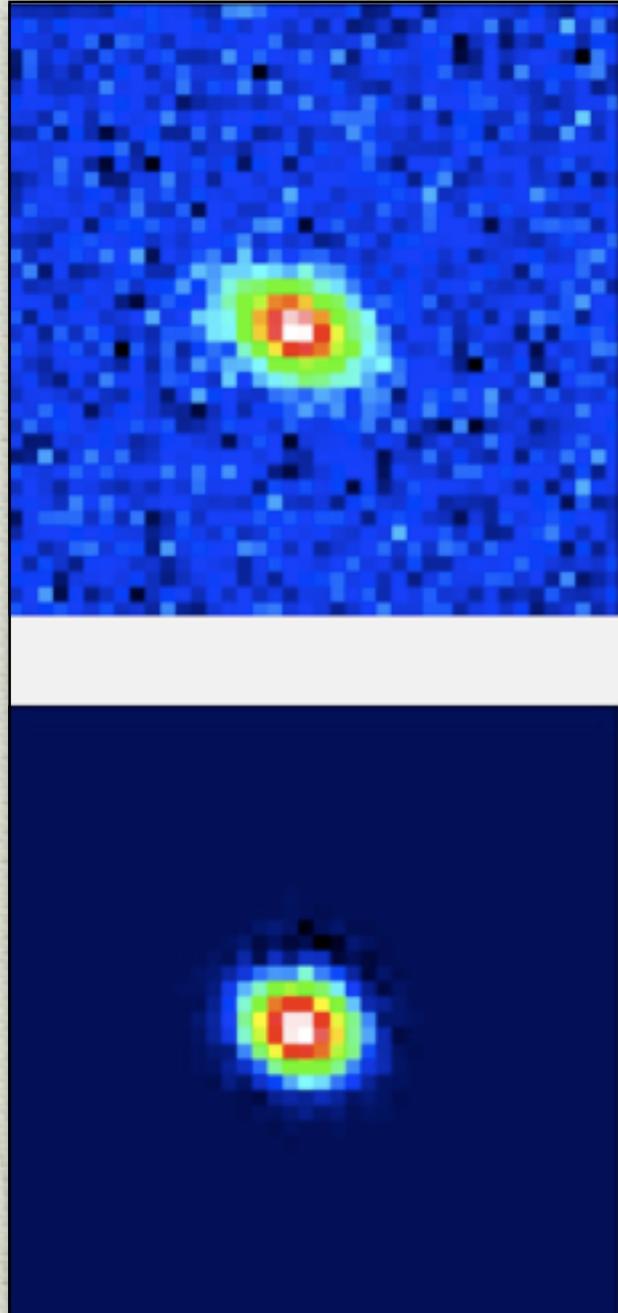
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$$\frac{(S/N)_\zeta}{(S/N)_\delta} = \sqrt{5} \frac{S_\zeta}{S_\delta}$$

# DEIMOS for flexion...

- Model independent as KSB, HOLICS
- It allows an **exact** PSF deconvolution as shapelets



$$G_{ijk}^{w,\star} = \int W(\vec{x}) I^c(\vec{x}) x_i x_j x_k d\vec{x}$$

DEWEIGHTING (real space)

$$G_{ijk}^{\star} = \int I^c(\vec{x}) x_i x_j x_k d\vec{x}$$

DECONVOLVING (moment space)

$$\{G^{\star}\}_{i,j} = \sum_k^i \sum_l^j \binom{i}{k} \binom{j}{l} \{G\}_{k,l} \{P\}_{i-k,j-l}$$

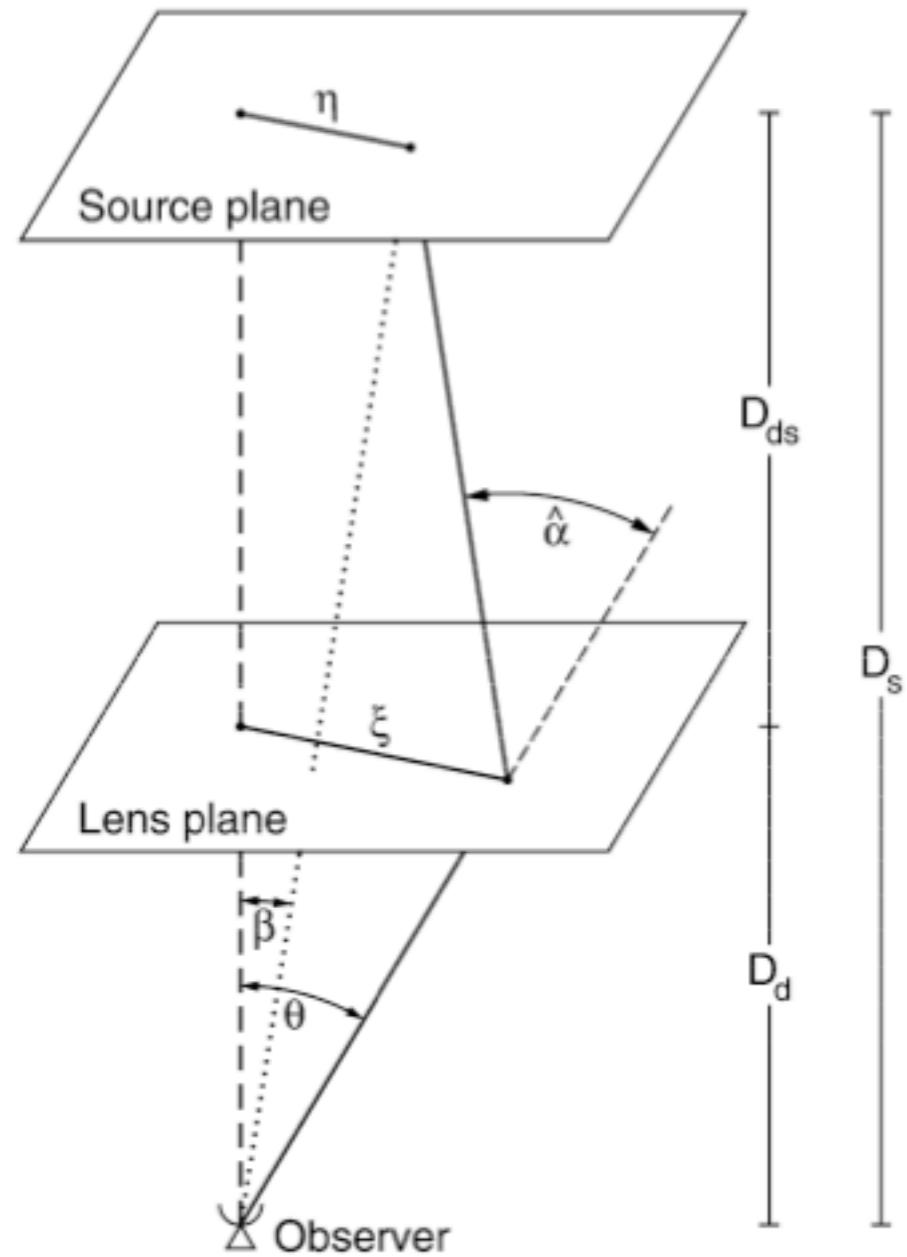
FORMING Spin-1 and Spin-3 estimators

Melchior, MV et al. 2011

# Conclusions

- ◆ Flexion probes the gravitational potential at small scales. Very useful for having extra info on cluster potential, substructures detection, halo's ellipticity...
- ◆ Measurements of spin-1 and spin-3 distortions DOES NOT mean a direct measurement of F and G flexion! [shear-flexion cross talk]
- ◆ Spin-1 and spin-3 distortions depend on the galaxy morphology: use this information to select your galaxies !
- ◆ Intrinsic flexion ?
- ◆ What is the optimal weighting function for measuring flexion ?
- ◆ New systematics ?

# Derivatives of the gravitational tidal field



$$\beta = \theta - \alpha(\theta)$$

It is a non linear equation!

$$\beta_i \simeq \theta_i - \Psi_{,ij}\theta_j - \frac{1}{2}\Psi_{,ijk}\theta_j\theta_k$$

$$\beta \simeq (1 - \kappa)\theta - \gamma\theta^* - \frac{1}{4}\mathcal{F}^*\theta^2 - \frac{1}{2}\mathcal{F}\theta\theta^* - \frac{1}{4}\mathcal{G}(\theta^*)^2$$

Second derivative  
of the gravitational  
potential

Third derivative of the gravitational  
potential