International Max Planck Research School for Astronomy & Cosmic Physics at the University of Heidelberg
IMPRS

Biases in, and correction to, KSB shear measurements

Viola et al. 2010 (arXiv:1006.2470)

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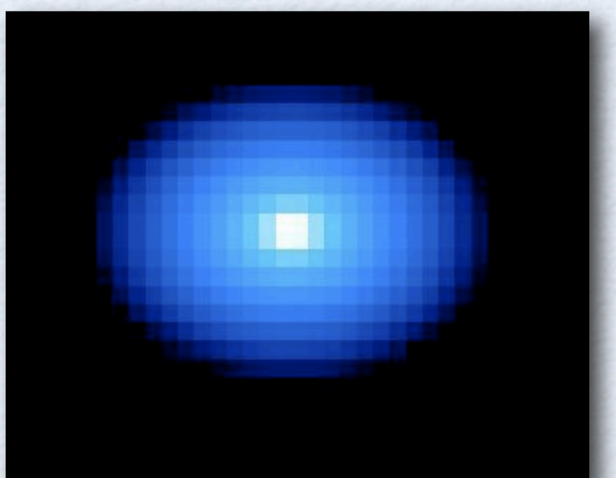
10 years of cosmic shear

From shape to ellipticity

Model indepent approach based on the moments of the source:

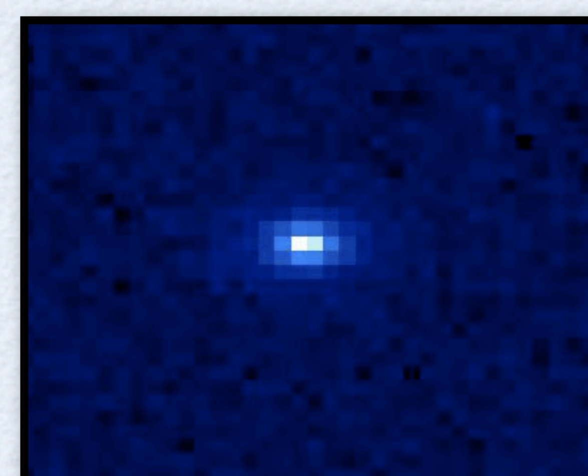
$$M_{ij...k} = \int I(\vec{\theta})\theta_i\theta_j...\theta_k d^2\theta$$



In theory:

$$\chi = \frac{(Q_{11} - Q_{22}) + 2iQ_{12}}{Q_{11} + Q_{22}}$$

In practise: real images are noisy!
Weighting function is required



KSB

$$\chi_\alpha - \chi^s_\alpha = g^\beta \Big[\underbrace{P^{sh}_{\alpha\beta}}_{\text{Shear-ellipticity relation}} + \underbrace{P^{sm}_{\alpha\gamma}\big(P^{sm,*\,-1}_{\gamma\delta} P^{sh,*}_{\delta\beta}\big)}_{\text{PSF correction}}\Big] + \underbrace{P^{sm}_{\alpha\beta}\, q_\beta}_{\text{PSF anistotropy correction}}$$

ASSUMPTIONS:

1) Strength of apparent distortion
2) Width of the weighting function
3) Mapping between convolved an unconvolved ellipticity
4) Ellipticity of the PSF

0. AVERAGING

$$0 = \langle \chi^s \rangle = \left\langle \frac{\chi - 2g + g^2\chi^*}{1 + |g|^2 - 2\Re(g\chi^*)} \right\rangle$$

Shear = average over galaxy's ellipticities

$$\neq$$

$$\langle \tilde{g} \rangle = \left\langle \frac{\chi(1 - \sqrt{1 - |\chi|^2})}{|\chi|^2} \right\rangle$$

Shear = average over shear's estimators

Difference depends on the moments of the intrinsic ellipticity distribution

Since the relation between g and χ is non linear shear estimation and average do not commute.

1. Strength of the apparent distortion

1. Shear is small (on average)

$$\chi_\alpha - \chi^s_\alpha = g^\beta P^{sh}_{\alpha\beta} = g^\beta(-2\chi_\alpha\chi_\beta + 2\delta_{\alpha\beta})$$

2. Definition of a shear estimator for a single object (is not small in general)

$$\tilde{g}^{KSB} \simeq \frac{\chi_1}{2} + \boxed{\frac{\chi_1^3}{2} + \frac{\chi_1^5}{2} + ...}$$

- If shear is not assumed to be small:

$$\tilde{g}^{KSB3} \simeq \frac{\chi_1}{2} + \frac{\chi_1^3}{8} + \frac{\chi_1^5}{16} + ...$$

- If P is equal to half of its trace:

$$\tilde{g}_1^{KSBtr} \simeq \frac{\chi_1}{2} + \boxed{\frac{\chi_1^3}{4} + \frac{\chi_1^5}{8} + ...}$$

No mathematical justification !

- If shear & ellipticity are assumed to be small:

$$\tilde{g}_1^{KSB1} = \frac{\chi_1}{2}$$

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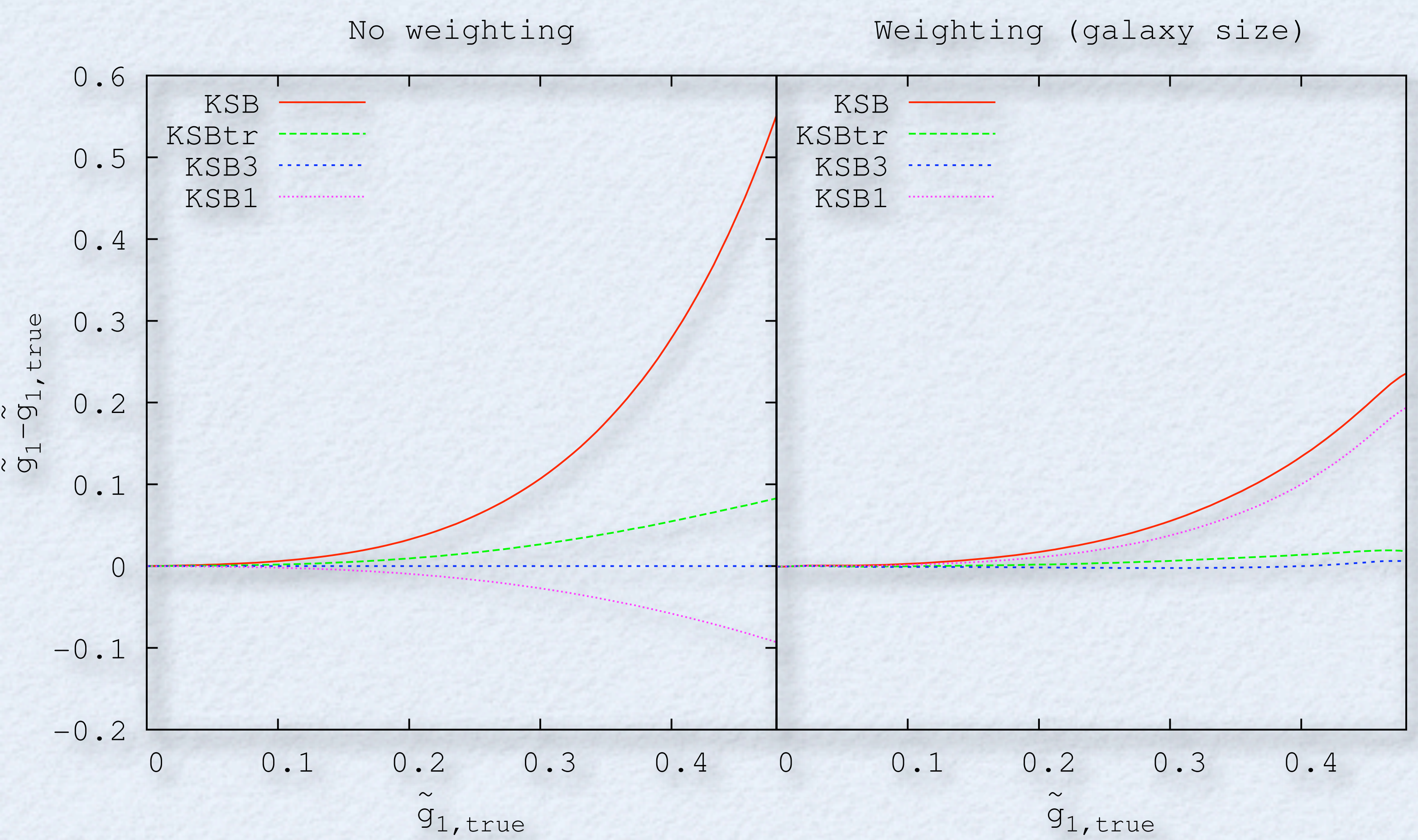
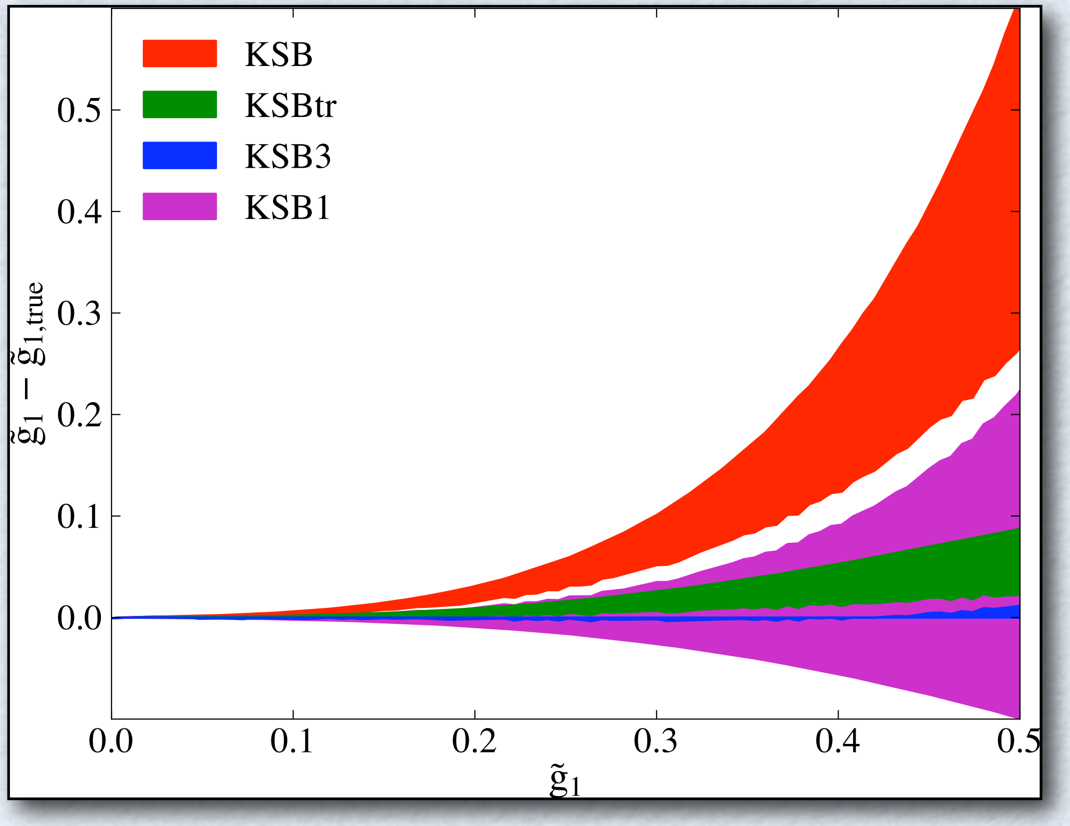
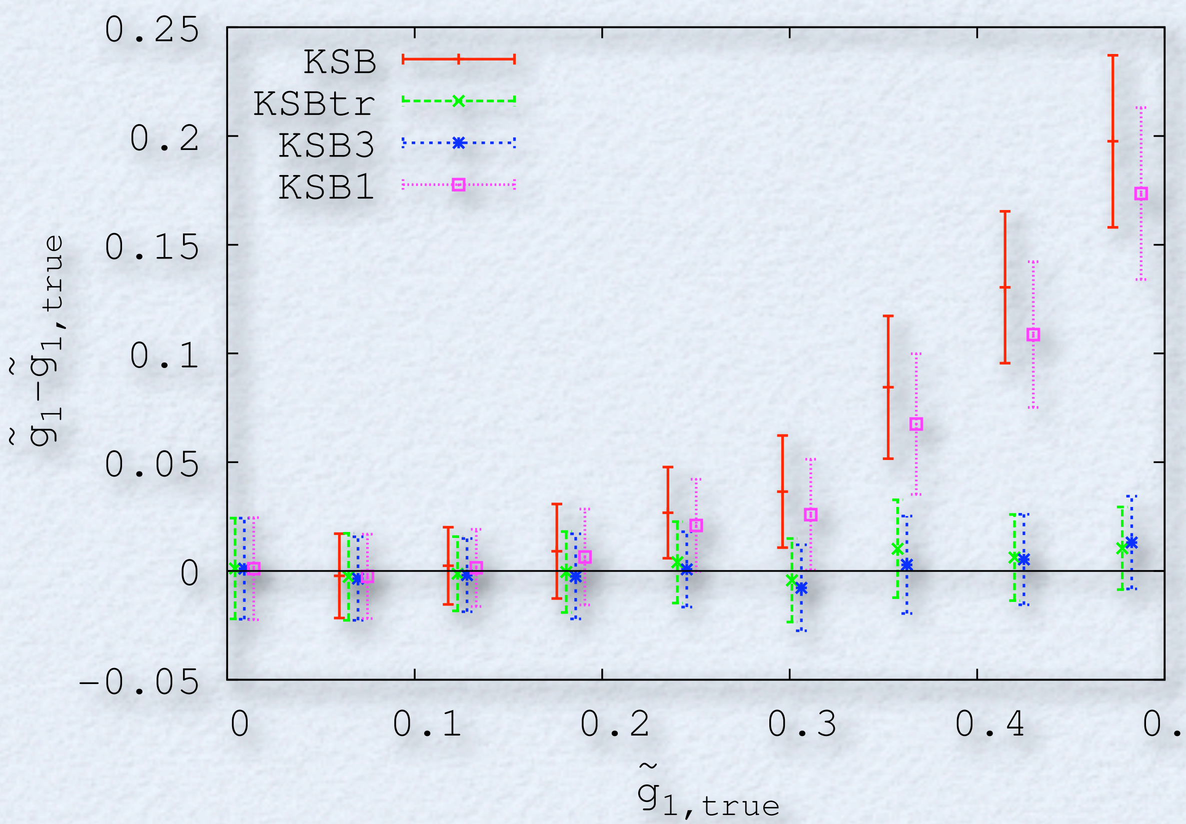
2. WIDTH OF THE WEIGHTING FUNCTION





Shear estimates are very dependent on the size of the weighting function (due to a poor description of the weighting process)

SOME TESTS



Noise test

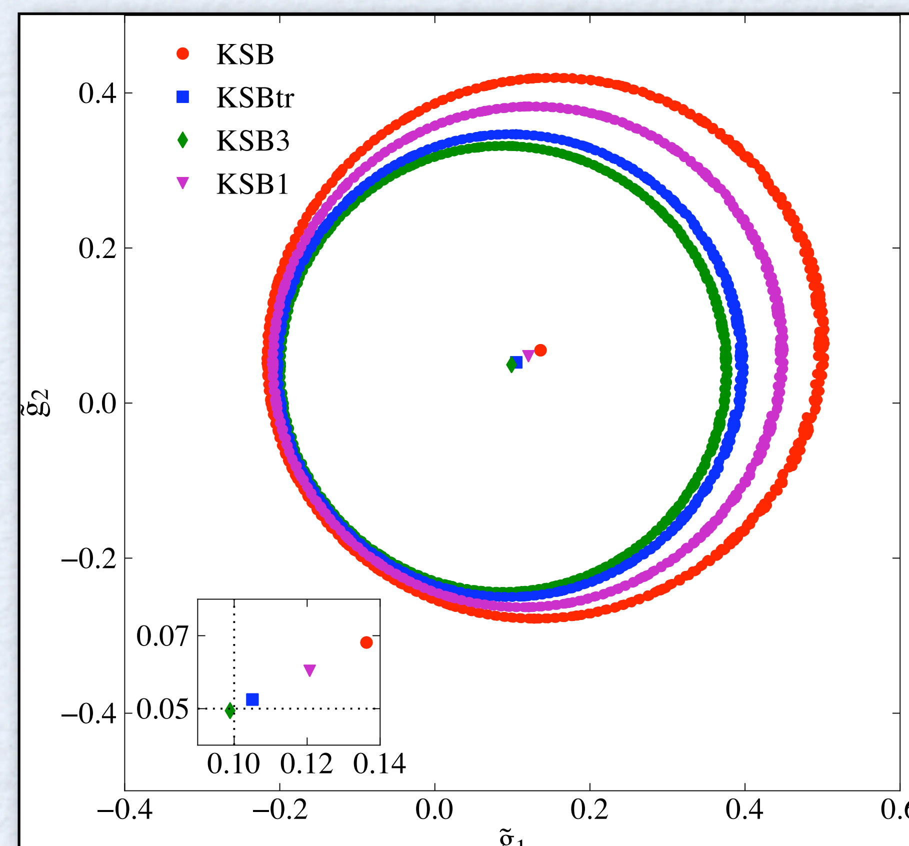
Rms = 1e-3

Number object = 100

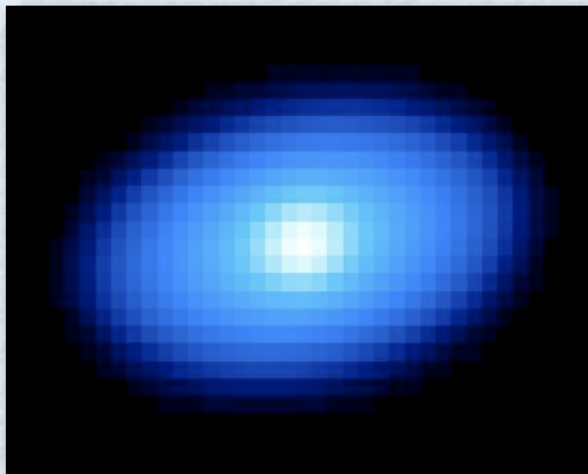
Ring test

Intrinsic ellipticity = 0.3

g=(0.1,0.05)



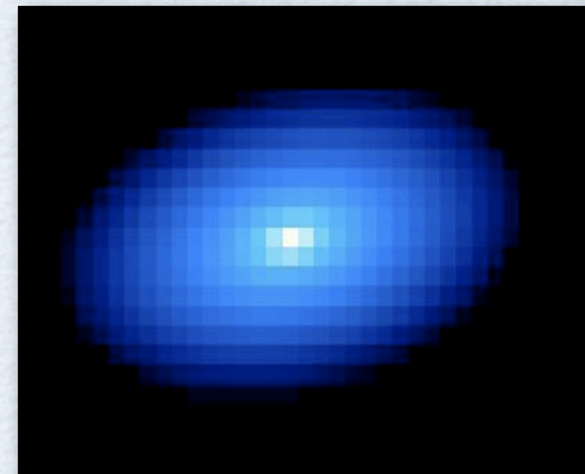
PSF: how should be treated...
(DEIMOS: Melchior et al. in prep)



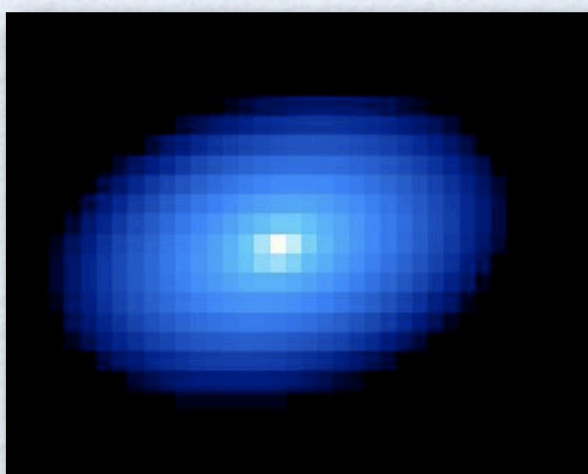
Observed plane

$$h(\chi^{obs}) = \tilde{\chi} = \frac{\chi^{obs}}{1-A}$$

$$A = \frac{M}{TrQ}\frac{TrQ^*}{M^*}$$



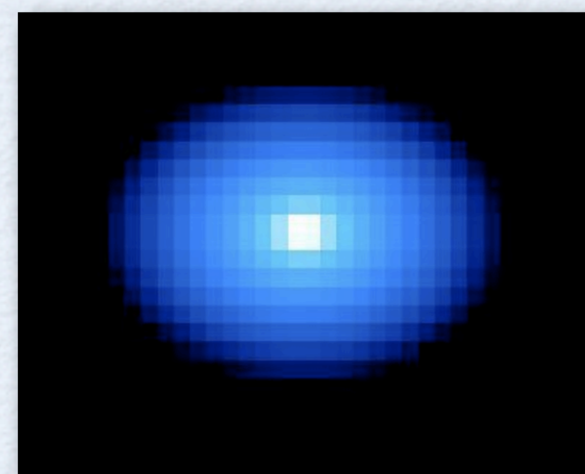
Lens plane



Lens plane

(on average)

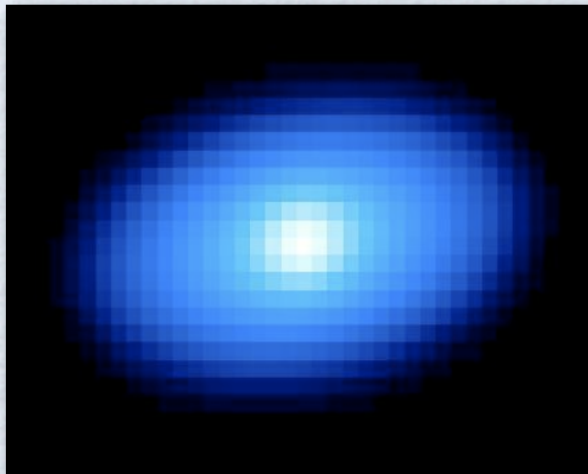
$$g \simeq \frac{\langle\tilde{\chi}\rangle}{2} + \ldots$$



Source plane

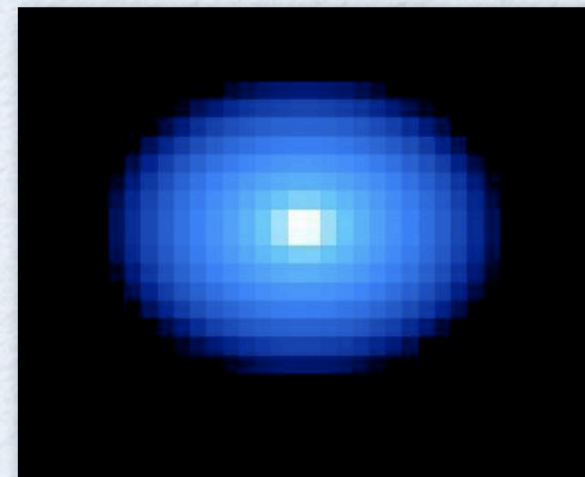
Deconvolve ---> Average ---> Compute shear

3. Mapping between convolved and unconvolved ellipticity



Lens plane

$$\chi_\alpha^{obs} = \chi_\alpha^{sh} - \chi_\alpha^{g}$$



Source plane

Mapping between shear and ellipticity (KSB,KSB1,KSBtr,KSB3)

$$\chi_\alpha^{sh} = P_{\alpha\beta}^{sh} g^\beta$$

+ PSF correction

$$\chi_\alpha^{g} = P_{\alpha\gamma}^{sm} (P^{sm,*})_{\gamma\beta}^{-1} \chi_\beta^{sh,*} \qquad P_{\alpha\beta}^{sm} = \frac{M}{TrQ}\delta_{\alpha\beta}$$

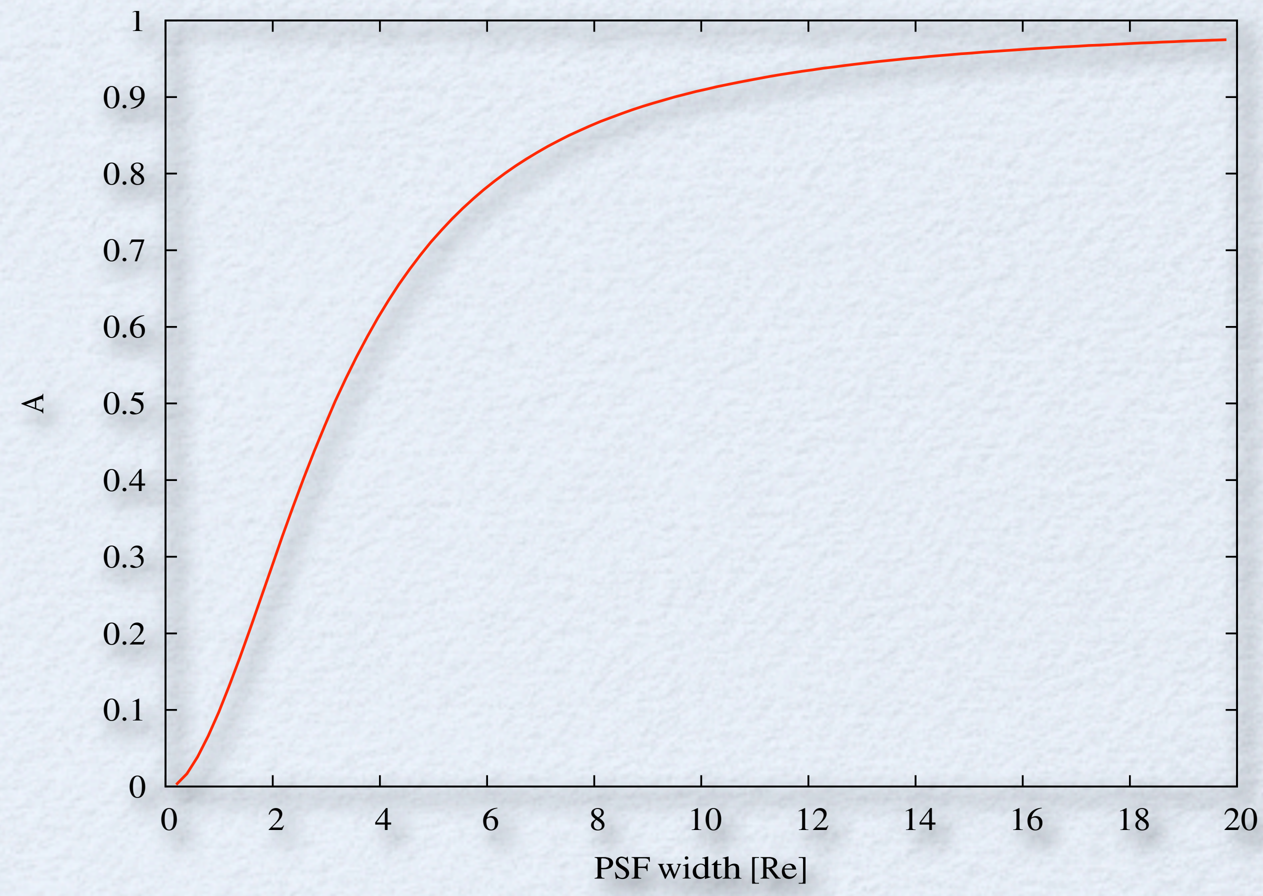
The “deconvolution” done by KSB is correct only if the unconvolved ellipticity is small (almost never the case) or if the PSF width vanishes

Effect of the PSF in the measurements

$$\boxed{\tilde{g} \simeq \frac{\tilde{\chi}}{2} + \frac{\tilde{\chi}^3}{8} + \mathcal{O}(\tilde{\chi}^5)}$$

Shear estimate if a proper PSF deconvolution is performed

$$\begin{aligned}
\tilde{g}^{KSB} &\simeq \frac{\tilde{\chi}}{2} + \frac{\tilde{\chi}^3}{2}(1-A) + \mathcal{O}(\tilde{\chi}^5) \\
\tilde{g}^{KSB1} &= \frac{\tilde{\chi}}{2} \\
\tilde{g}^{KSBtr} &\simeq \frac{\tilde{\chi}}{2} + \frac{\tilde{\chi}^3}{4}(1-A) + \mathcal{O}(\tilde{\chi}^5) \\
\tilde{g}^{KSB3} &\simeq \frac{\tilde{\chi}}{2} + \frac{\tilde{\chi}^3}{8}(1-4A) + \mathcal{O}(\tilde{\chi}^5)
\end{aligned}$$



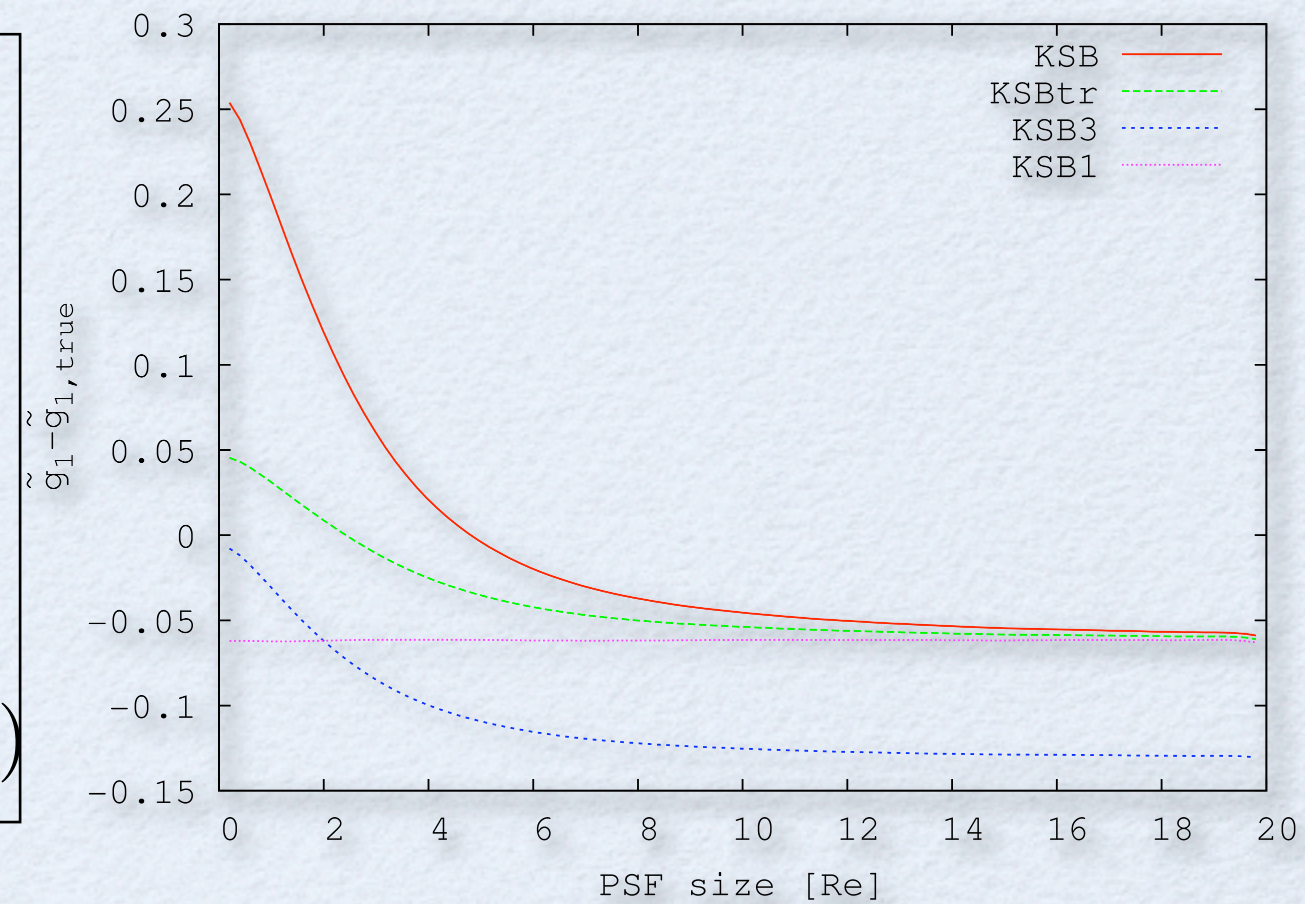
Shear estimates depend on the width of the PSF (via A)

Effect of the PSF in the measurements

$$\boxed{g \simeq \frac{\tilde{\chi}}{2} + \frac{\tilde{\chi}^3}{8} + \mathcal{O}(\tilde{\chi}^5)}$$

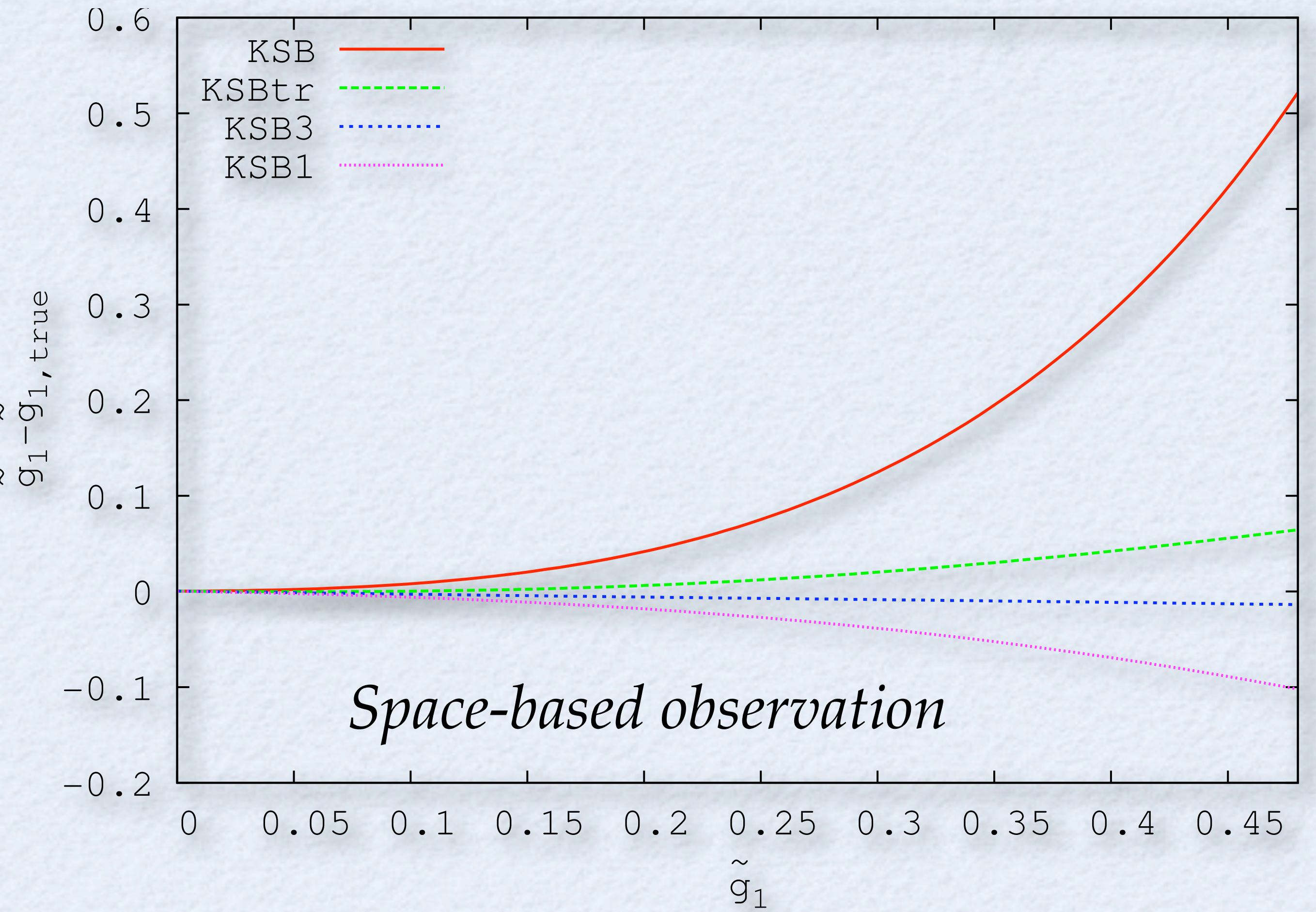
Shear estimate if a proper PSF deconvolution is performed

$$\begin{aligned} g^{KSB} &\simeq \frac{\tilde{\chi}}{2} + \frac{\tilde{\chi}^3}{2}(1-A) + \mathcal{O}(\tilde{\chi}^5) \\ g^{KSB1} &= \frac{\tilde{\chi}}{2} \\ g^{KSBtr} &\simeq \frac{\tilde{\chi}}{2} + \frac{\tilde{\chi}^3}{4}(1-A) + \mathcal{O}(\tilde{\chi}^5) \\ g^{KSB3} &\simeq \frac{\tilde{\chi}}{2} + \frac{\tilde{\chi}^3}{8}(1-4A) + \mathcal{O}(\tilde{\chi}^5) \end{aligned}$$

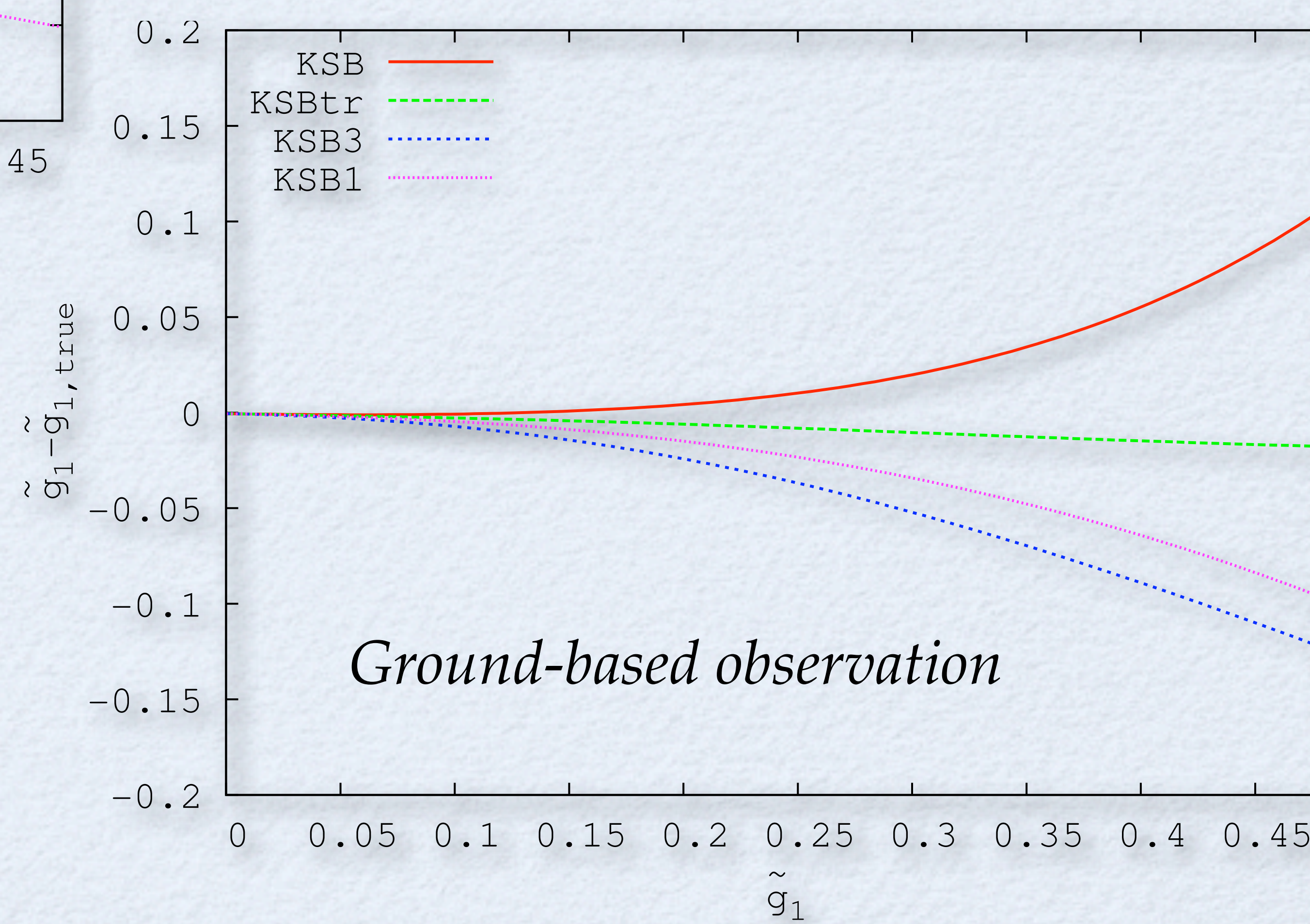


Shear estimates depend on the width of the PSF (via A)

SOME TESTS



Wrong approximation of the PSF deconvolution has the effect of biasing low the estimator



Problems in KSB

- Shear estimates are averaged instead of ellipticities.
- Individual shear estimates are computed under the assumption that the shear is small, but that's true only AFTER averaging ---> Overestimation
- PSF deconvolution is only approximated, and the approximation holds only for objects which are almost circular or for vanishing PSF. ---> Underestimation
- Anisotropies in the PSF can be proper correct only if they are VERY small

Assumptions made in KSB derivation and application are fulfilled only if the dispertion of the **intrinsic ellipticity** of galaxies **vanishes** and the **shear** is **small**. Moreover the **PSF** must be very narrow.

OUTLOOK

Melchior, Viola, Schäffer, Bartelmann 2010 (to appear soon!)

DEIMOS:

- Model independent (moment based)

- Perform an exact PSF deconvolution in moment space

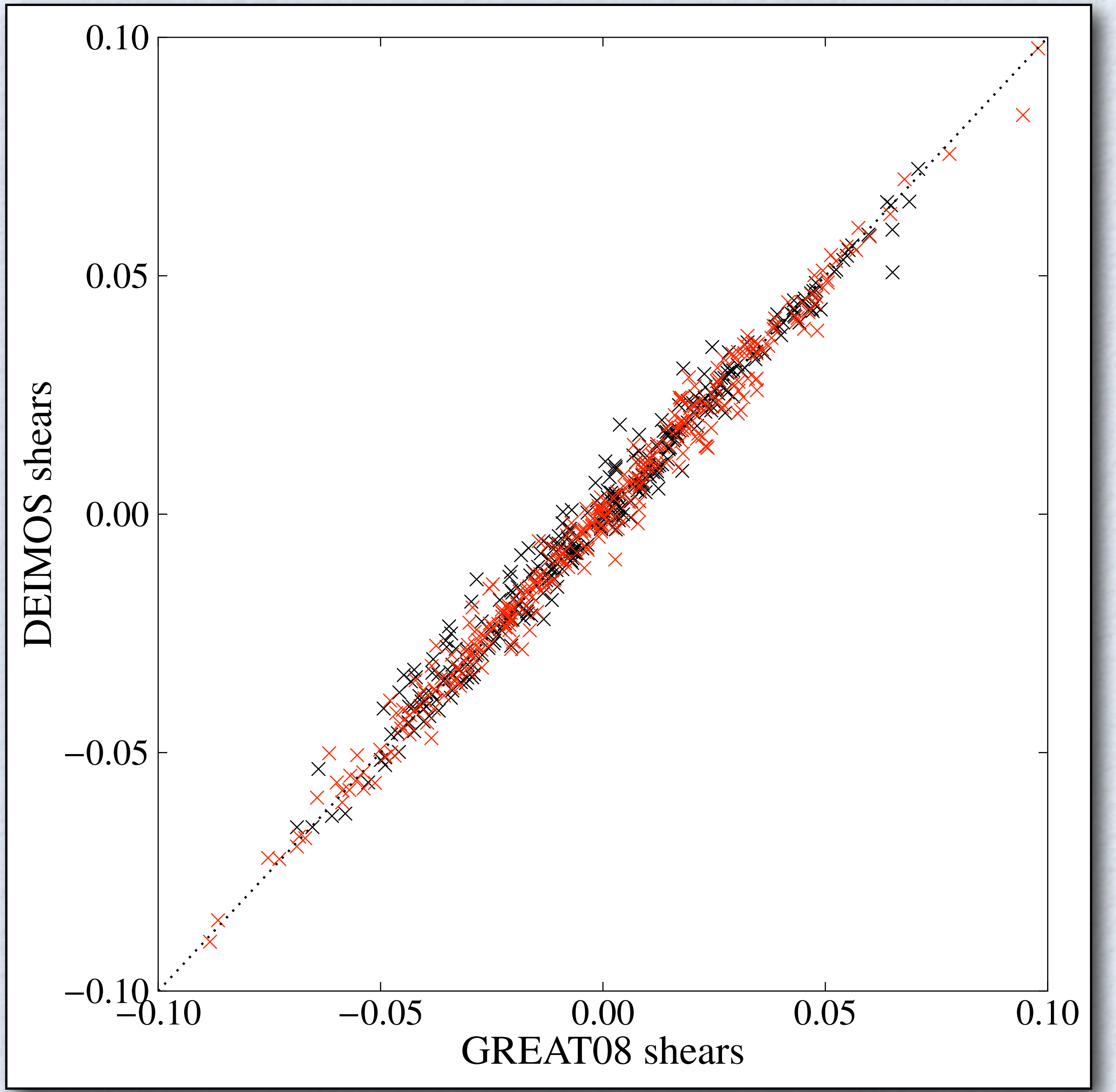
- Does not rely on any assumption about shape of the PSF

$$
\begin{aligned}
\{g\}_{0,0} &= \{g^\star\}_{0,0} \\
\{g\}_{0,1} &= \{g^\star\}_{0,1} - \{g\}_{0,0}\ \{p\}_{0,1} \\
\{g\}_{1,0} &= \{g^\star\}_{1,0} - \{g\}_{0,0}\ \{p\}_{1,0} \\
\{g\}_{0,2} &= \{g^\star\}_{0,2} - \{g\}_{0,0}\ \{p\}_{0,2} - 2\{g\}_{0,1}\ \{p\}_{0,1} \\
\{g\}_{1,1} &= \{g^\star\}_{1,1} - \tfrac{1}{2}\{g\}_{0,0}\ \{p\}_{1,1} - \{g\}_{0,1}\ \{p\}_{1,0} - \{g\}_{1,0}\ \{p\}_{0,1} \\
\{g\}_{2,0} &= \{g^\star\}_{2,0} - \{g\}_{0,0}\ \{p\}_{2,0} - 2\{g\}_{1,0}\ \{p\}_{1,0}
\end{aligned}
$$

OUTLOOK

Melchior, Viola, Schäffer, Bartelmann 2010 (to appear soon!)

DEIMOS:



300 sets of GREAT08RealNoise_Known

1. Strength of the apparent distortion

KSB : shear is small BUT ellipticity is not small....

$$\chi_\alpha - \chi^s_\alpha = g^\beta P^{sh}_{\alpha\beta} = g^\beta(-2\chi_\alpha\chi_\beta + 2\delta_{\alpha\beta})$$

$$\tilde{g}^{KSB} \simeq \frac{\chi_1}{2} + \frac{\chi_1^3}{2} + \frac{\chi_1^5}{2} + ...$$

Valid only at I order...BUT...ellipticity is not small!

KSBtr : shear is small, ellipticity is not small and P is equal to half of its trace

$$\tilde{g}_1^{KSBtr} \simeq \frac{\chi_1}{2} + \frac{\chi_1^3}{4} + \frac{\chi_1^5}{8} + ...$$

No mathematical justification !

KSB1 : shear is small and ellipticity is small

$$\tilde{g}_1^{KSB1} = \frac{\chi_1}{2}$$

Valid only at first order..BUT...ellipticity is not small!

KSB3 : shear and ellipticity are not assumed to be small...

$$\tilde{g}^{KSB3} \simeq \frac{\chi_1}{2} + \frac{\chi_1^3}{8} + \frac{\chi_1^5}{16} + ...$$