3D Dark Matter Mapping: A New Approach to Weak Lensing Tomography

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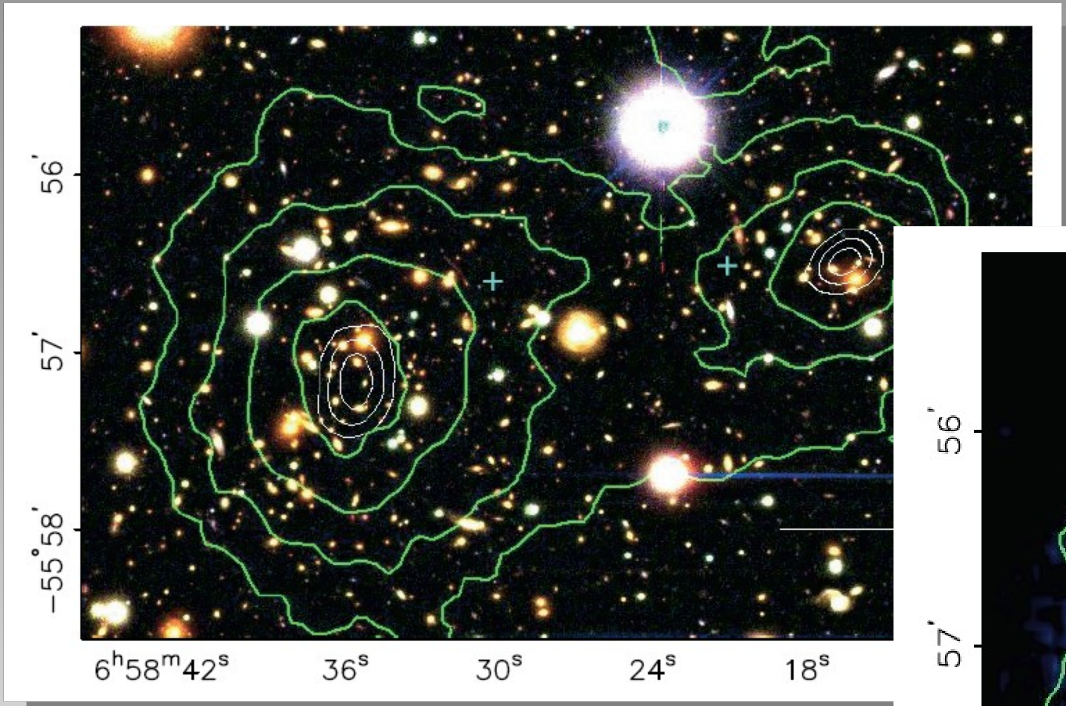
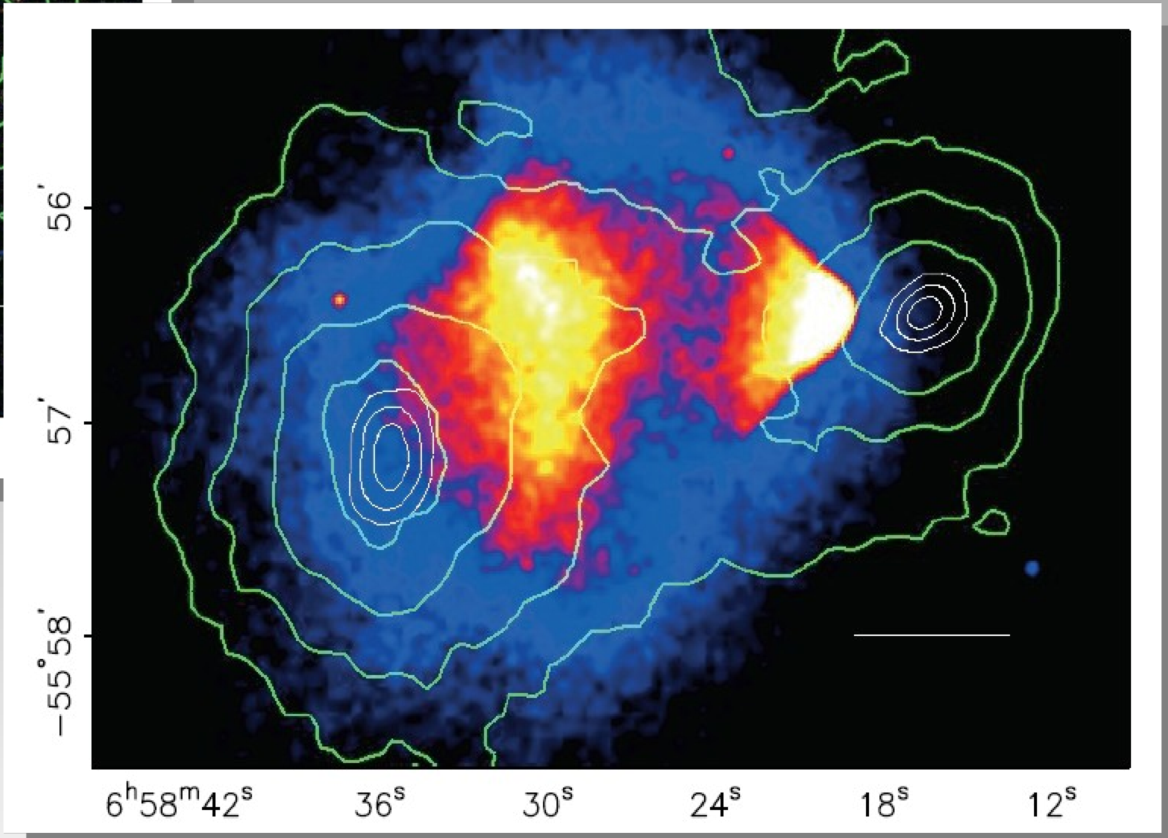
Classic Weak Lensing: 2D projection



No extraction of line-of-sight information

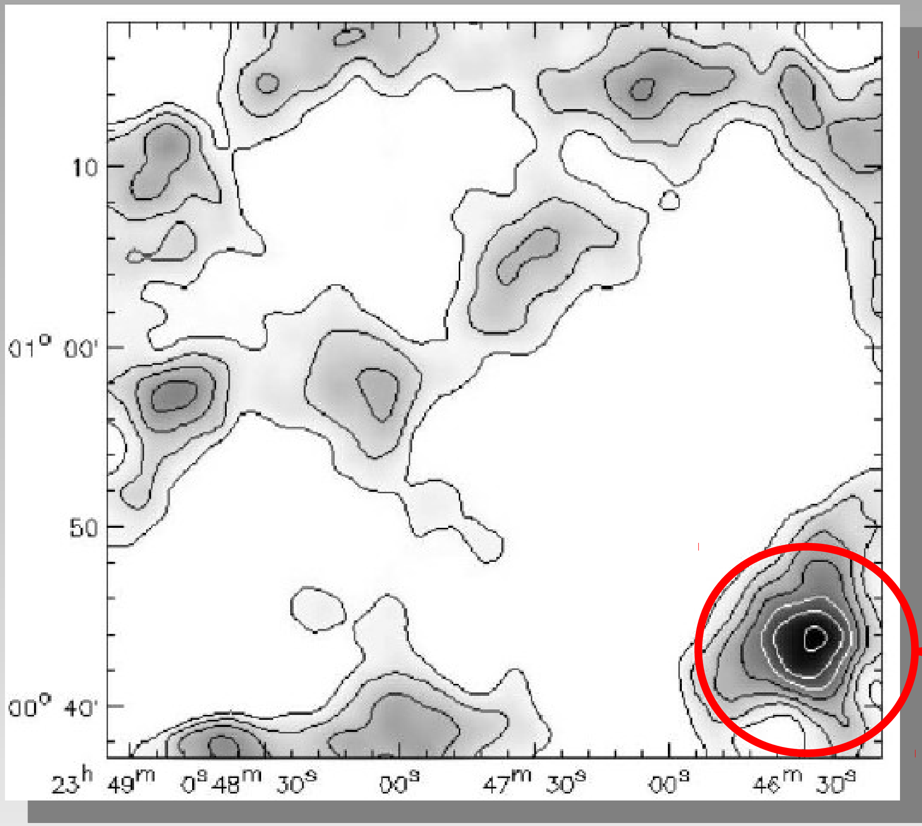


Clowe *et al.* 2006

Moving toward 3D

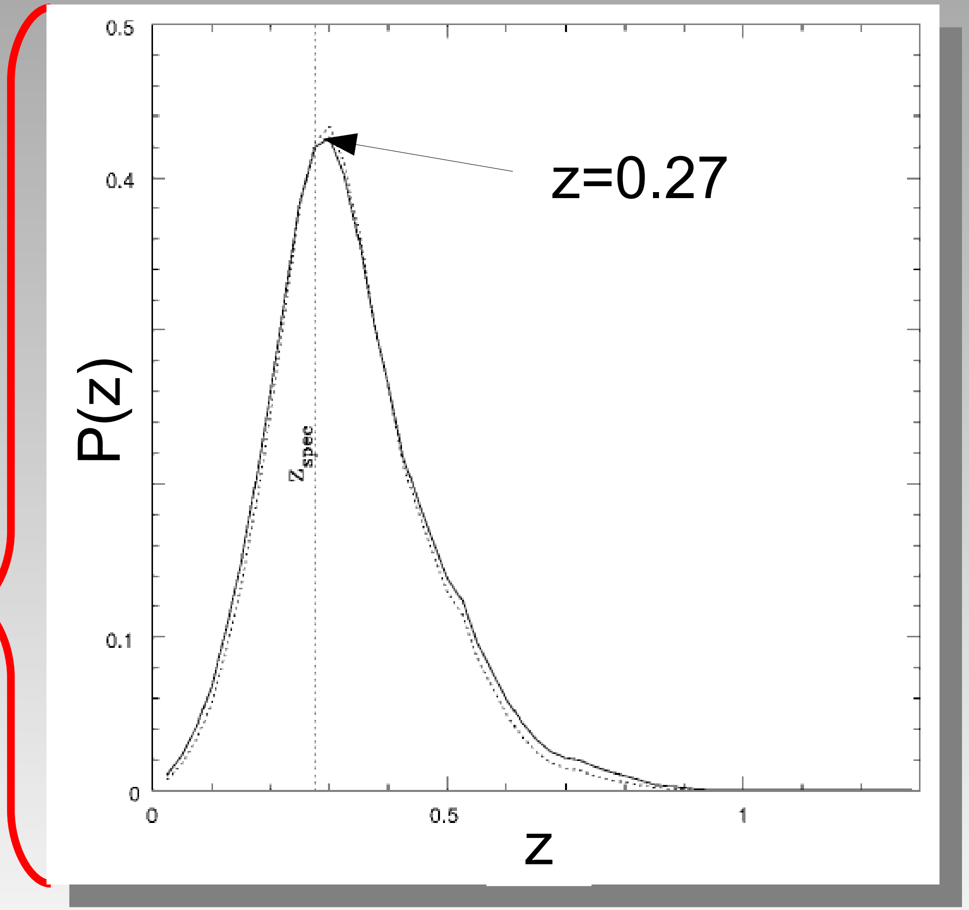
Parametric Methods

e.g. Wittman *et al.* 2001



Data: CTIO

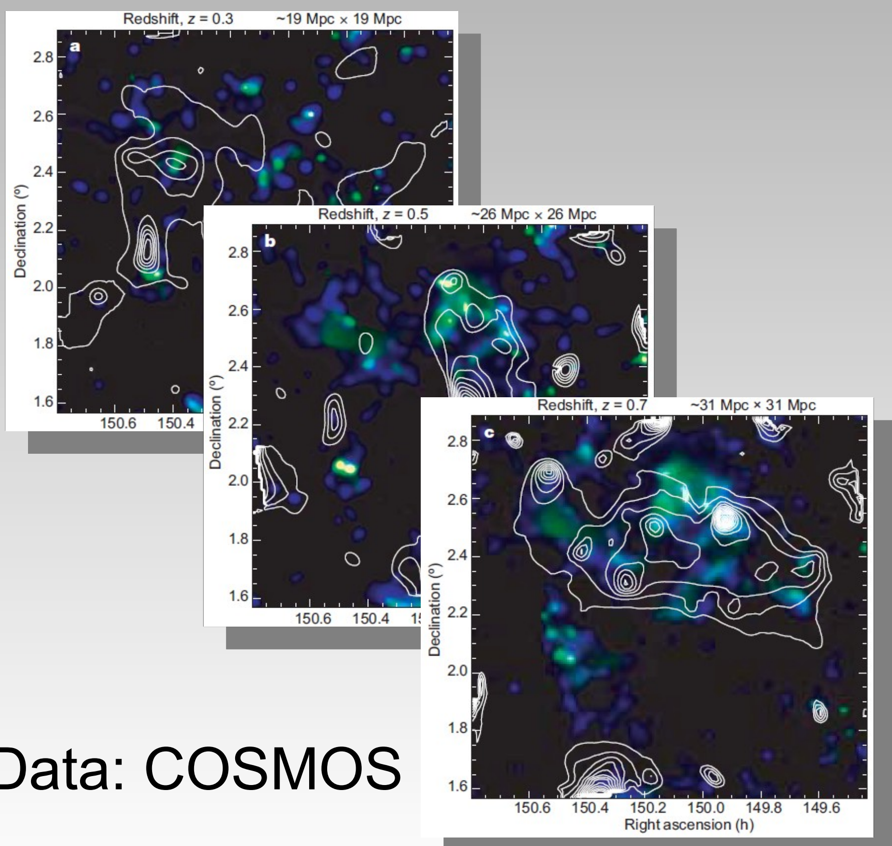
Fitted SIS and NFW profiles at different redshifts



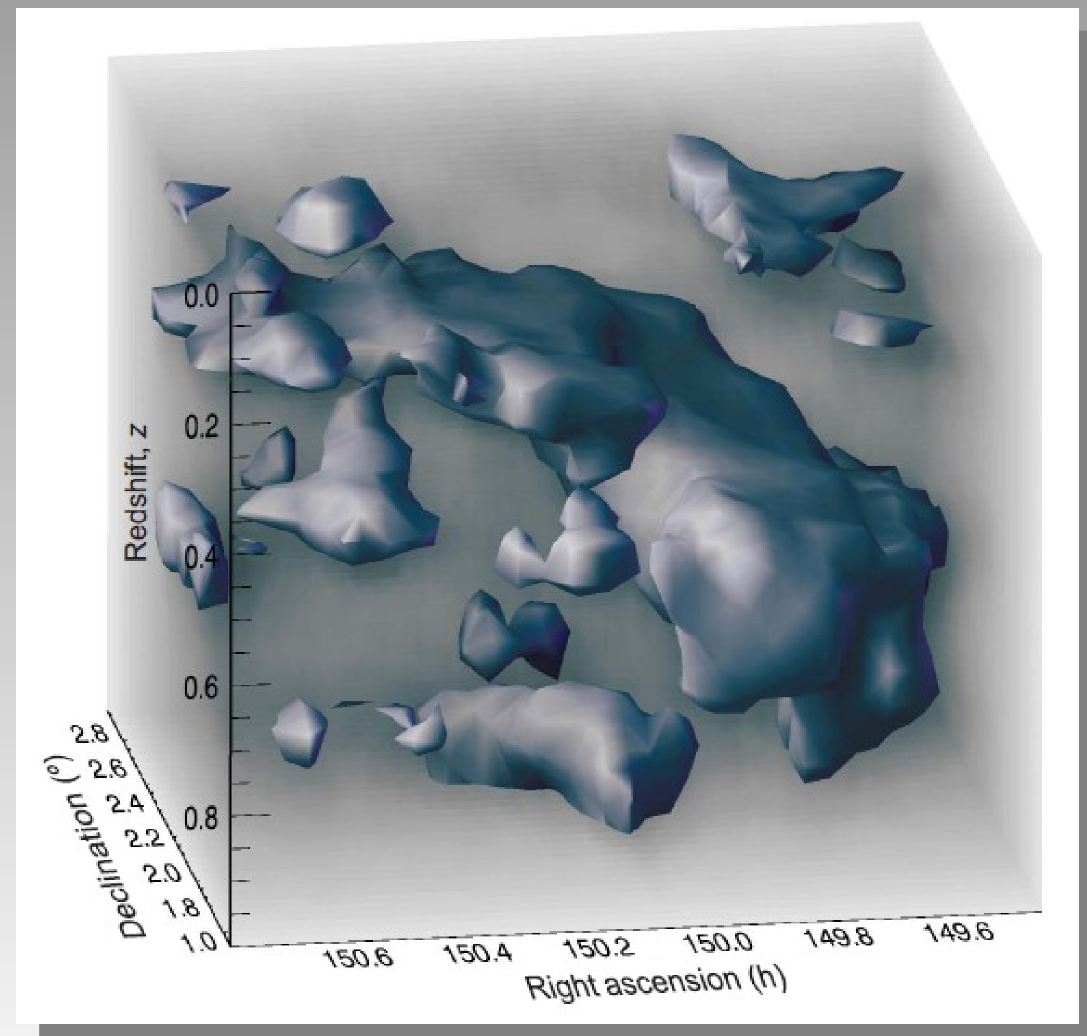
Moving toward 3D

Non-parametric “$2^{1}/_{2}$D” reconstruction

3D representation from three source-planes



Data: COSMOS



Massey *et al.* 2007

Toward a full 3D reconstruction:

$\gamma \rightarrow \kappa$:
$$\gamma(\vec{\theta}) = \int d\vec{\theta}'^{2} \kappa(\vec{\theta}') D(\vec{\theta} - \vec{\theta}')$$

$\vec{\gamma} = P_{\gamma\kappa} \vec{\kappa}$: operates in each source plane

$\kappa \rightarrow \delta$:
$$\kappa(\chi_S) = \frac{3 H_0^2 \Omega_M}{2} \int_0^{\chi_S} \frac{\chi(\chi_S - \chi)}{\chi_S} \frac{1 + \delta(\chi)}{a(\chi)} d\chi$$

$\vec{\kappa} = Q_{\kappa\delta} \vec{\delta}$: operates in each line-of-sight

Final Result: $\longrightarrow$ $\vec{\gamma} = M \vec{\delta}$

Linear Mapping

Given a noisy measurement γ, we want to solve for δ:

$$\vec{\gamma} = M\,\vec{\delta} + \vec{n}_\gamma$$

Best estimator is due to Aitken (1935):

$$\left(N_{\gamma\gamma} \equiv \langle \vec{n}_\gamma \vec{n}_\gamma{}^T \rangle \right)$$

$$\hat{\delta} = \left(M^T N_{\gamma\gamma}^{-1} M \right)^{-1} M^T N_{\gamma\gamma}^{-1} \vec{\gamma}$$

Problem: Noise can obscure the signal by several orders of magnitude!

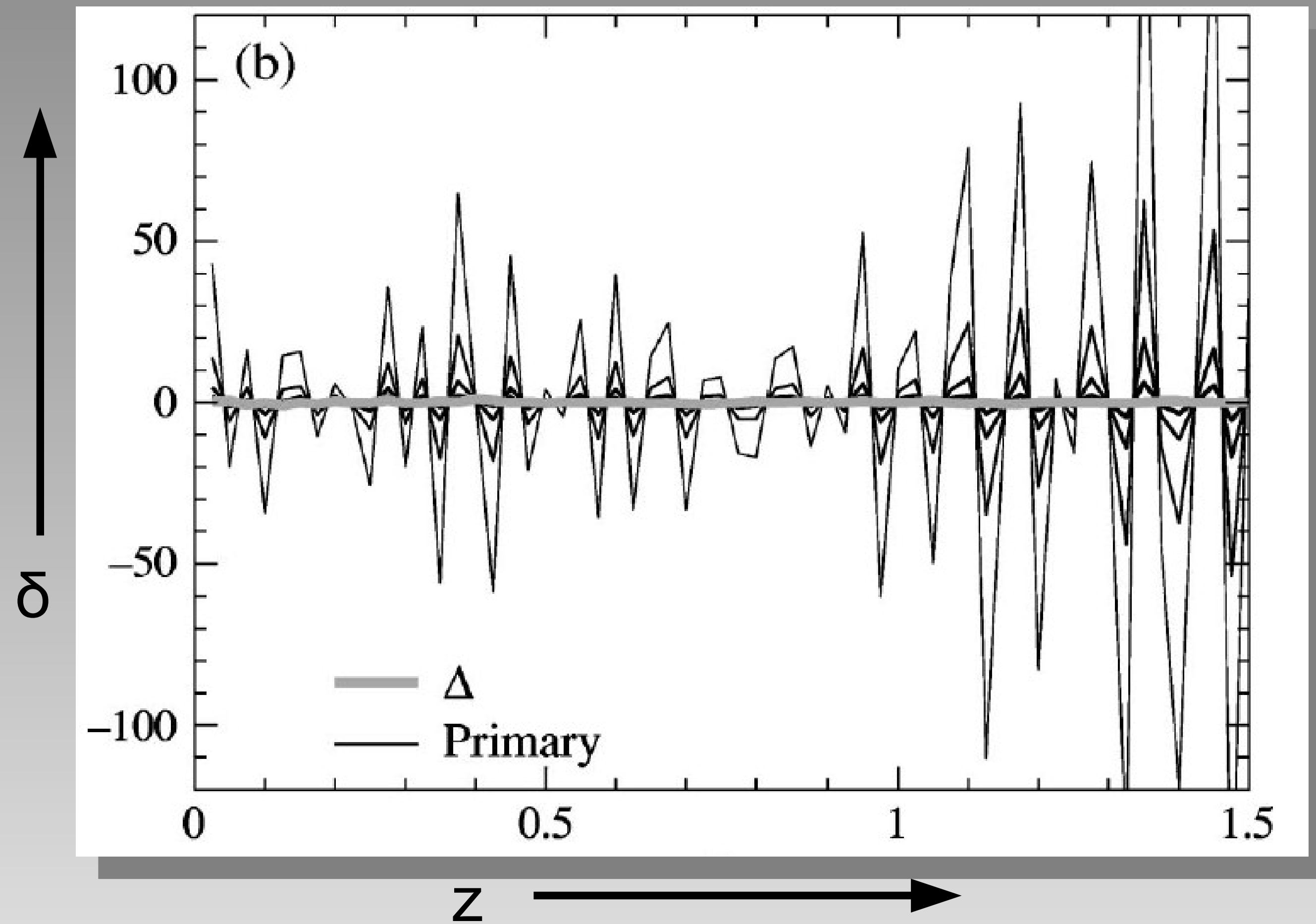
(Hu & Keeton 2002)

The Problem: Shear is too noisy

Hu & Keeton 2002:

Different lines:
Factors of 10
in noise

(Note: δ < -1
is unphysical)



Aitken estimator is no
good for noisy shear data.

A Solution: Wiener Filtering

Add a penalty to the χ^2 which suppresses large oscillations:

$$\chi^2 \rightarrow \chi^2 + H$$

Key results:

- successful in suppressing noise
- leads to a bias and spread in lens redshift
- requires NL power spectrum as input
- requires a relatively slow iterative solution

Hu&Keeton 2002, Simon *et al.* 2009

Can we do better?

$$\vec{\gamma} = M\,\vec{\delta} + n_\gamma \qquad \hat{\delta} = \left(M^T N_{\gamma\gamma}^{-1} M\right)^{-1} M^T N_{\gamma\gamma}^{-1} \vec{\gamma}$$

Singular Value Decomposition (SVD)

$$N_{\gamma\gamma}^{-1/2} M = U\,\Sigma\,V^T \qquad U^T U = V^T V = I$$

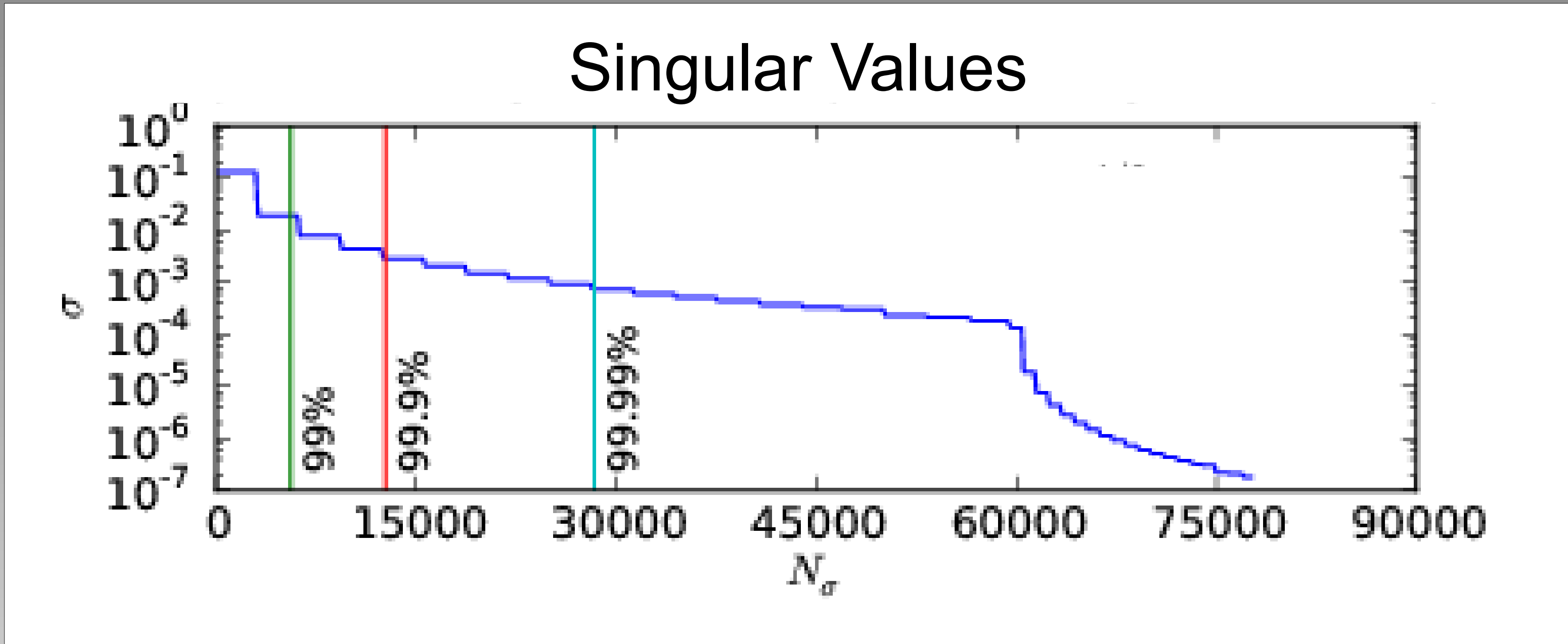
$\Sigma =$ diagonal

Aitken estimator becomes:

$$\hat{\delta} = V\,\Sigma^{-1}\,U^T N_{\gamma\gamma}^{-1/2}\,\vec{\gamma}$$

Small singular values lead to large noise in δ!

99.99% of variance is in less than 1/3 the SVD!



Standard trick:

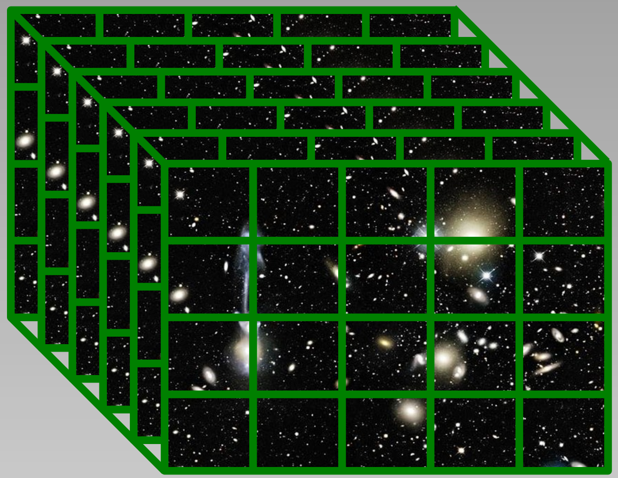
truncate the small singular values:

$$U\,\Sigma\,V^T \rightarrow \widetilde{U}\,\widetilde{\Sigma}\,\widetilde{V}^T$$

The Challenge...

For present-day surveys:

128x128 pixels



20 redshift bins

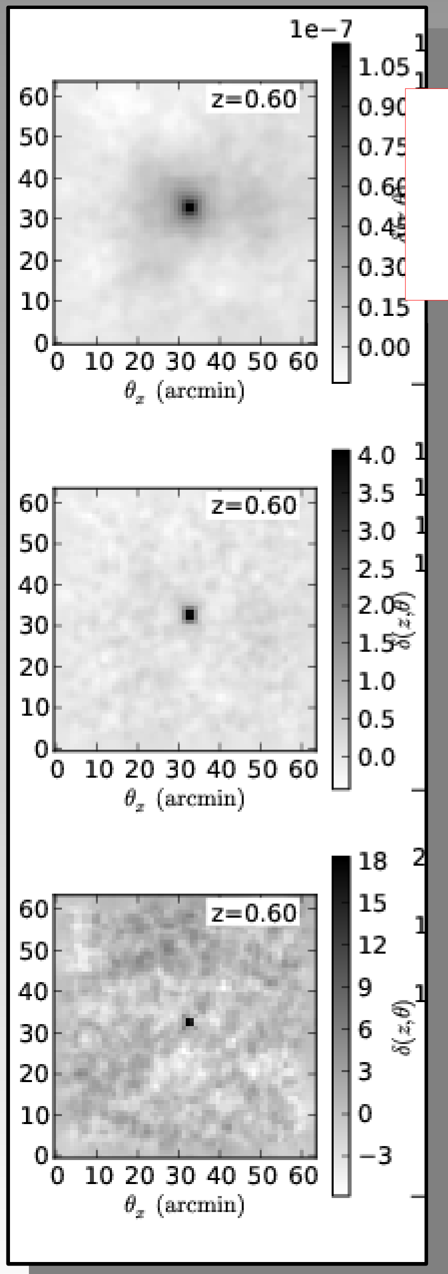
Matrix contains
1.3×10^{11} elements!
~2TB in memory!

SVD is non-trivial

Solution: tensor decomposition,
and a few reasonable approximations
(details in our upcoming paper)

Preliminary Results

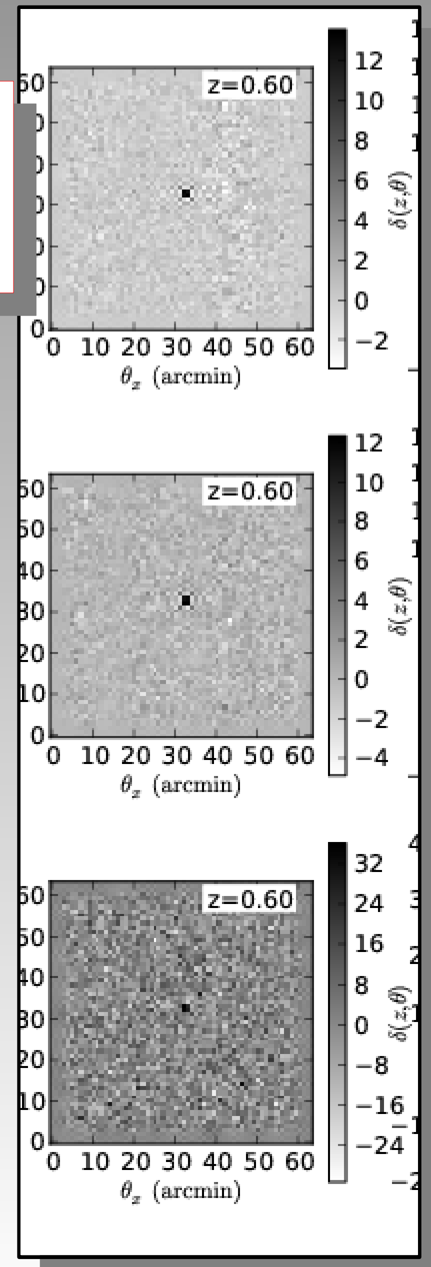
Wiener filter vs. SVD filter



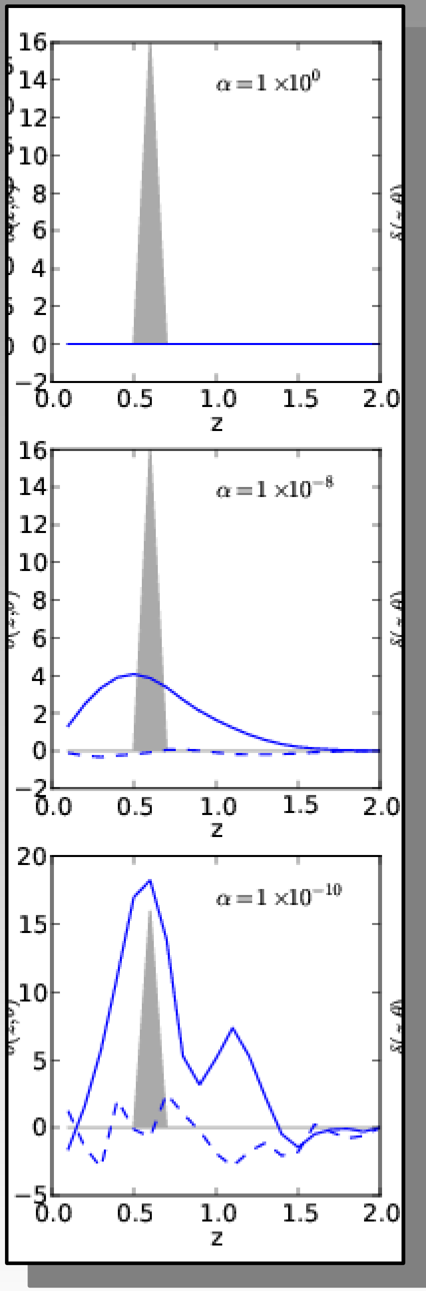
SVD shows less angular spread

More filtered

Less filtered

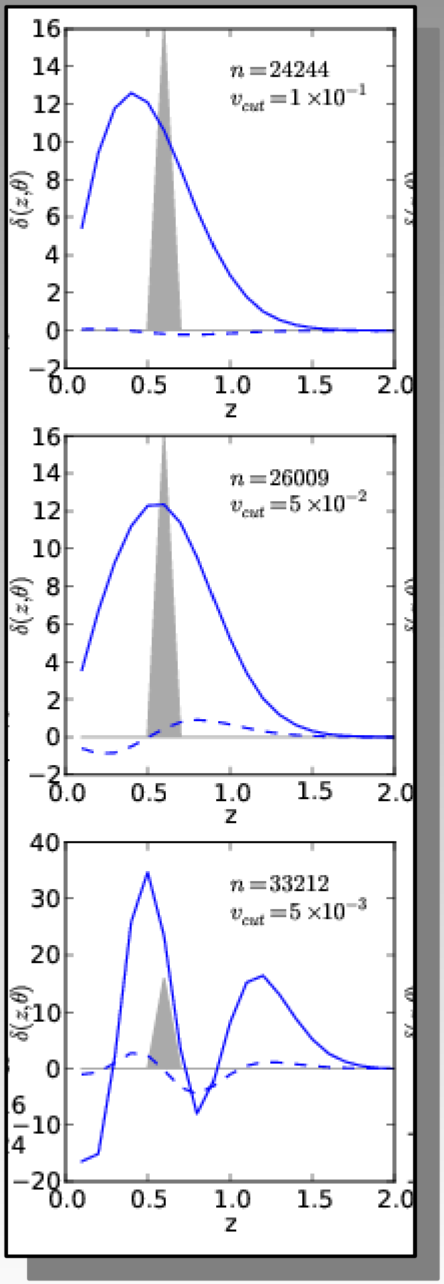


Wiener filter vs. SVD filter

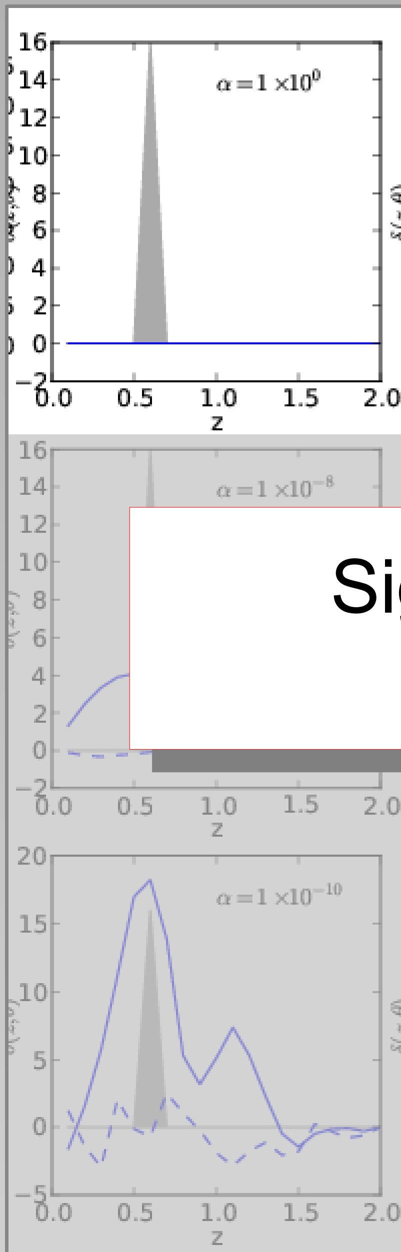


More filtered

Less filtered



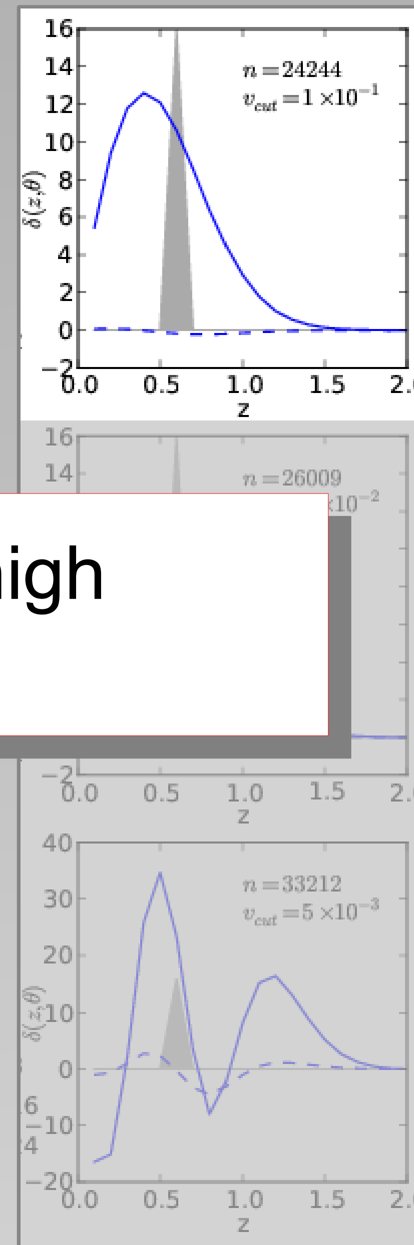
Wiener filter vs. SVD filter



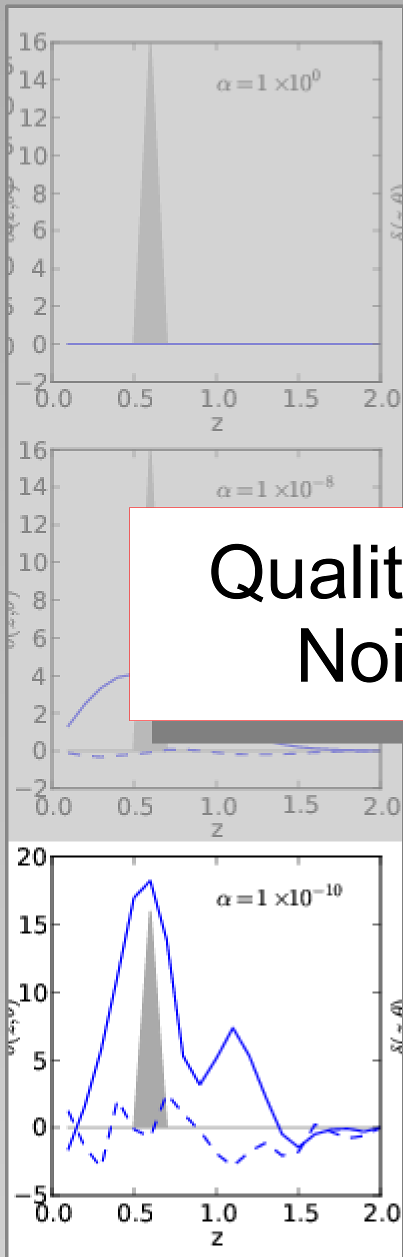
More filtered

Signal suppression for high Wiener filtering

Less filtered



Wiener filter vs. SVD filter



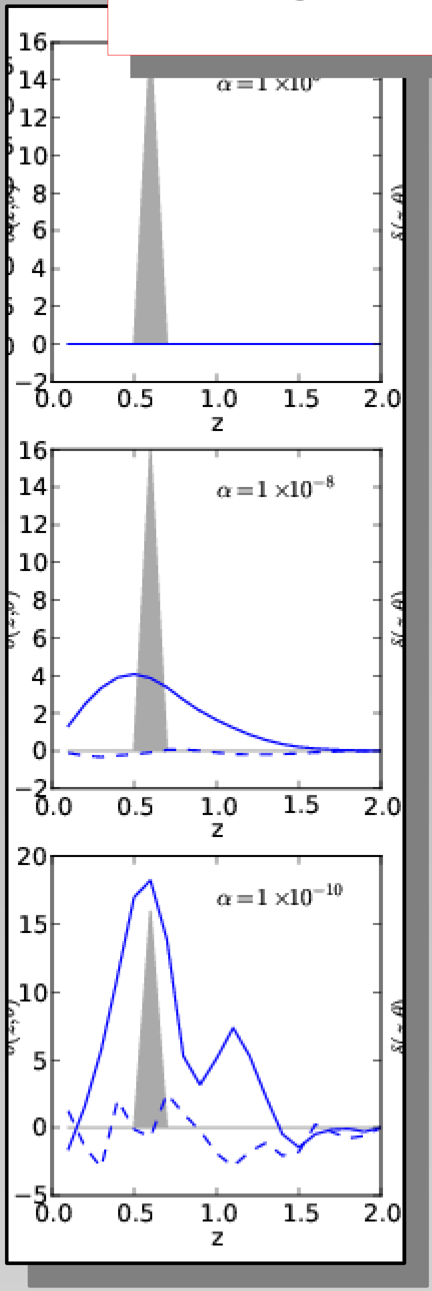
More filtered

Qualitatively similar for low filtering:
Noise induces spurious peaks

Less filtered

Wiener filter vs SVD filter

SVD: faster computation time:



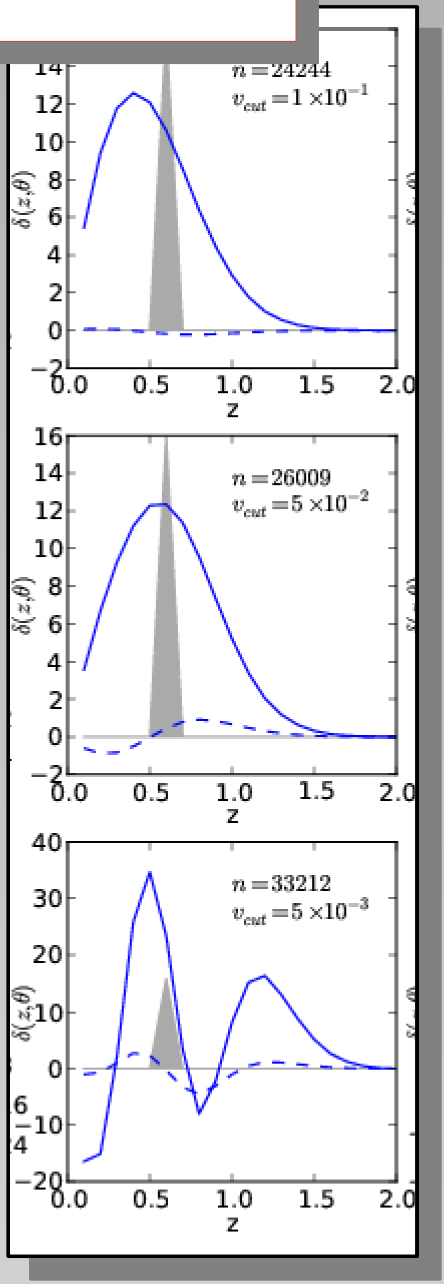
~10x

More filtered

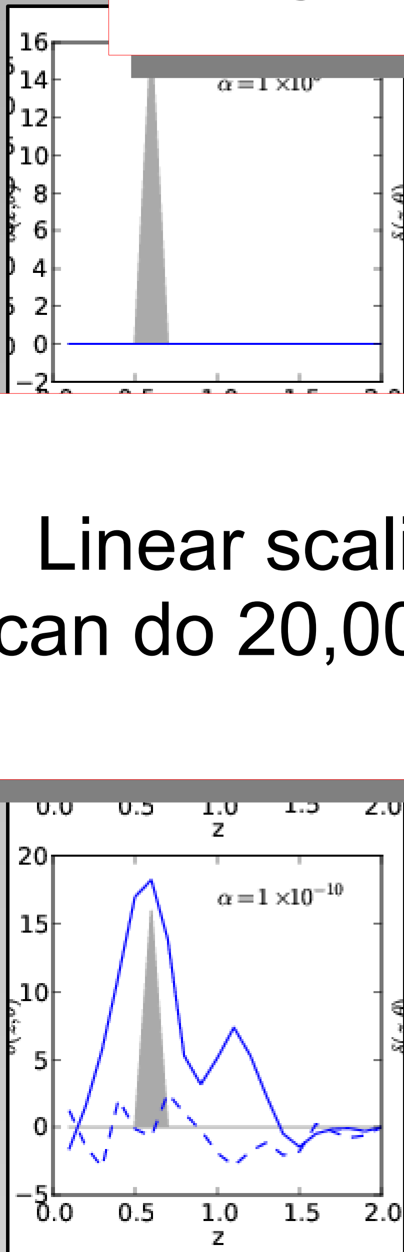
~100x

Less filtered

~1000x



SVD: faster computation time:



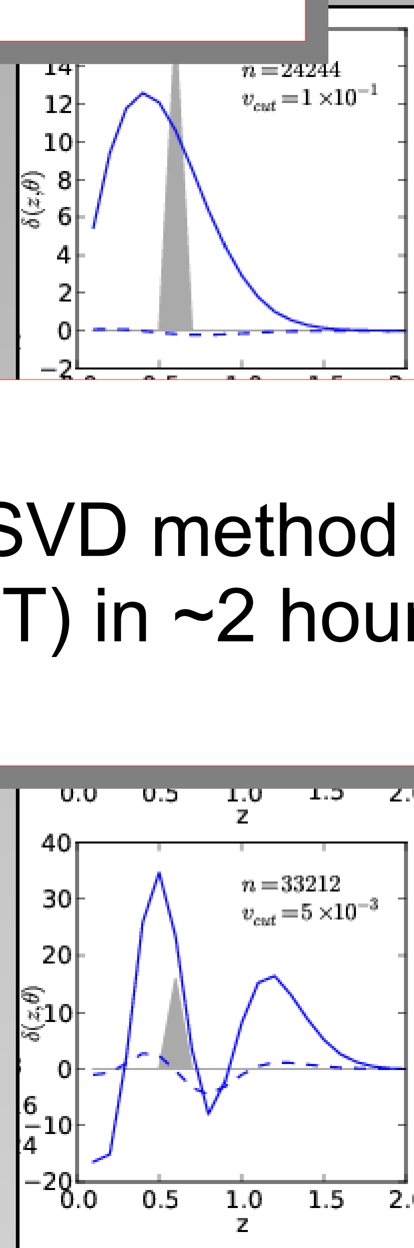
~10x

More filtered

Linear scaling with survey area: SVD method can do 20,000 deg^2 field (e.g. LSST) in ~2 hours

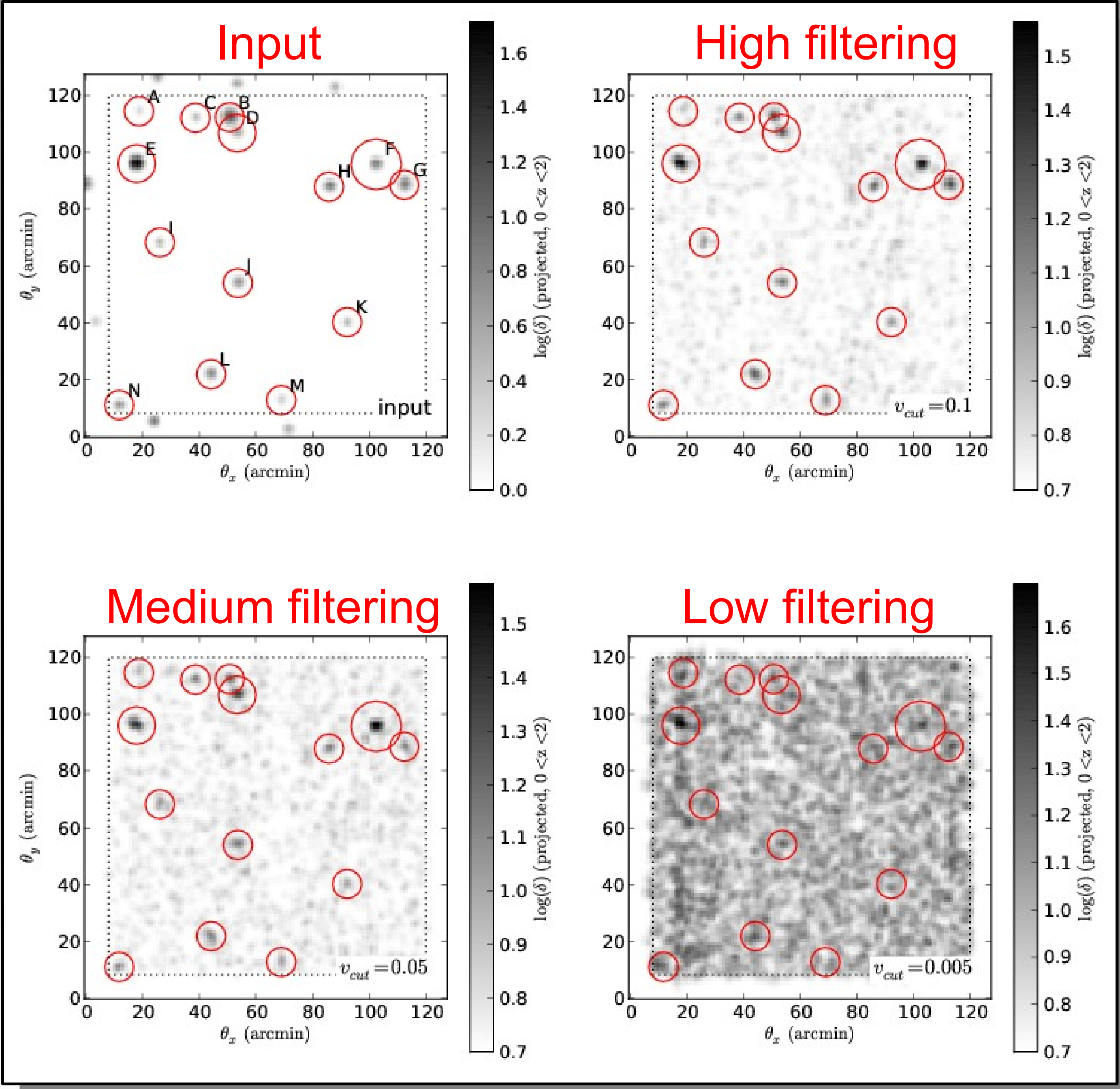
Less filtered

~1000x



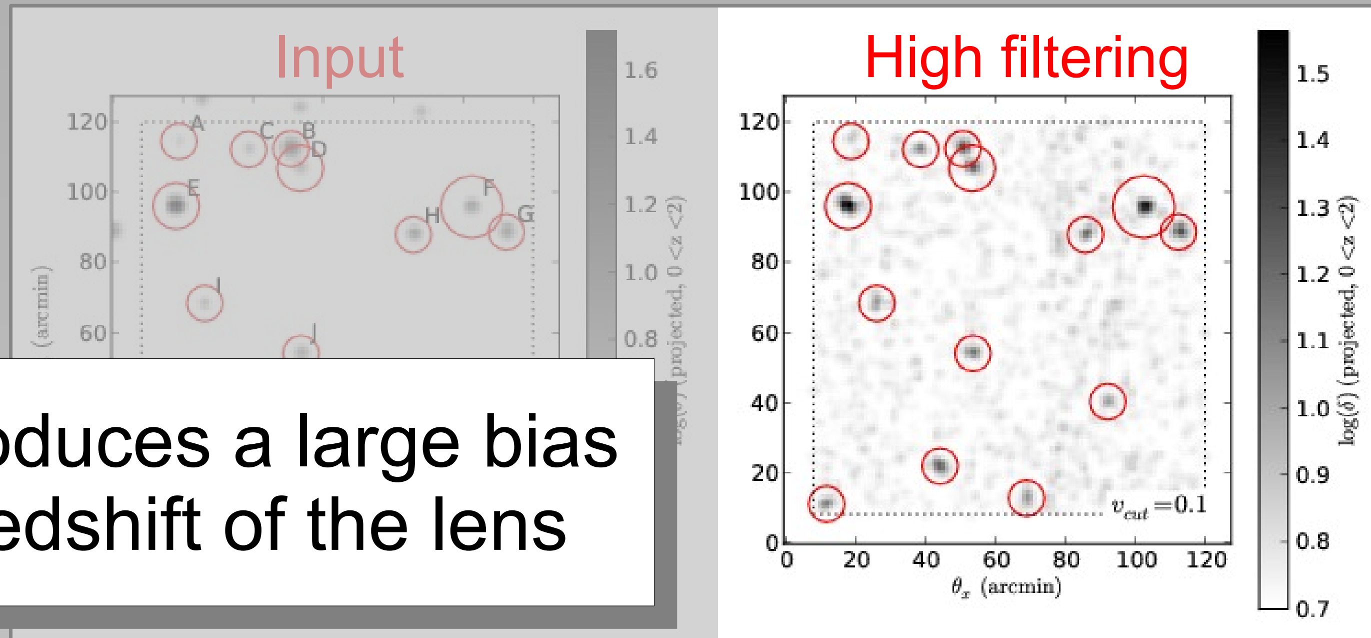
Angular Resolution and Redshift Bias

(SVD filter)

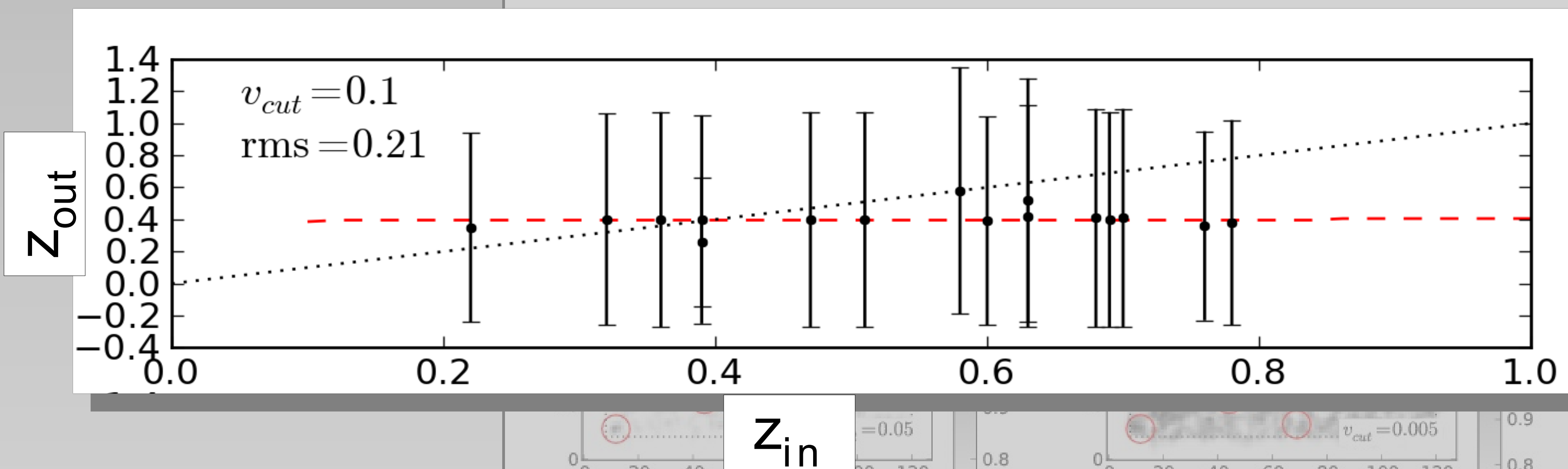


Angular Resolution and Redshift Bias

(SVD filter)

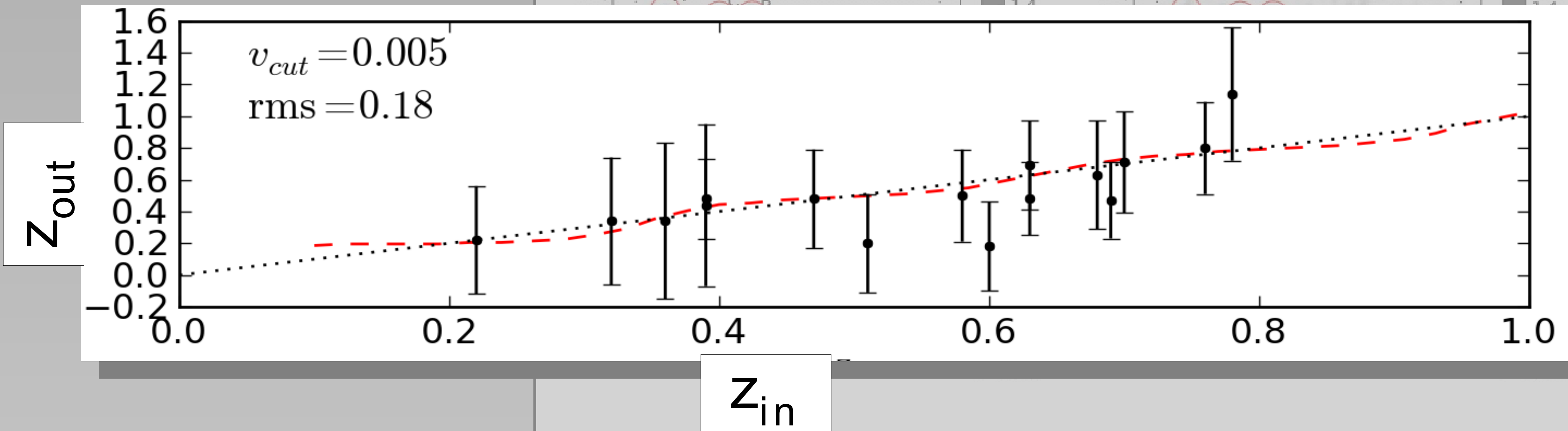


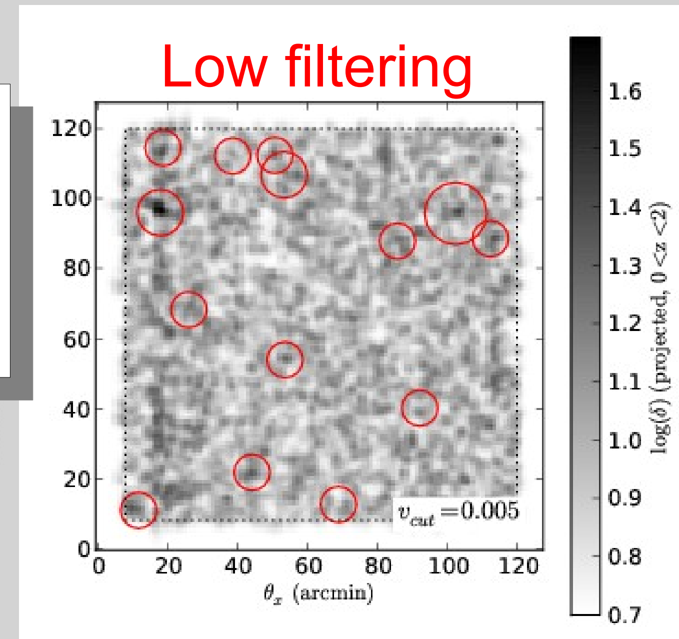
High filtering introduces a large bias and spread in redshift of the lens



Angular Resolution and Redshift Bias

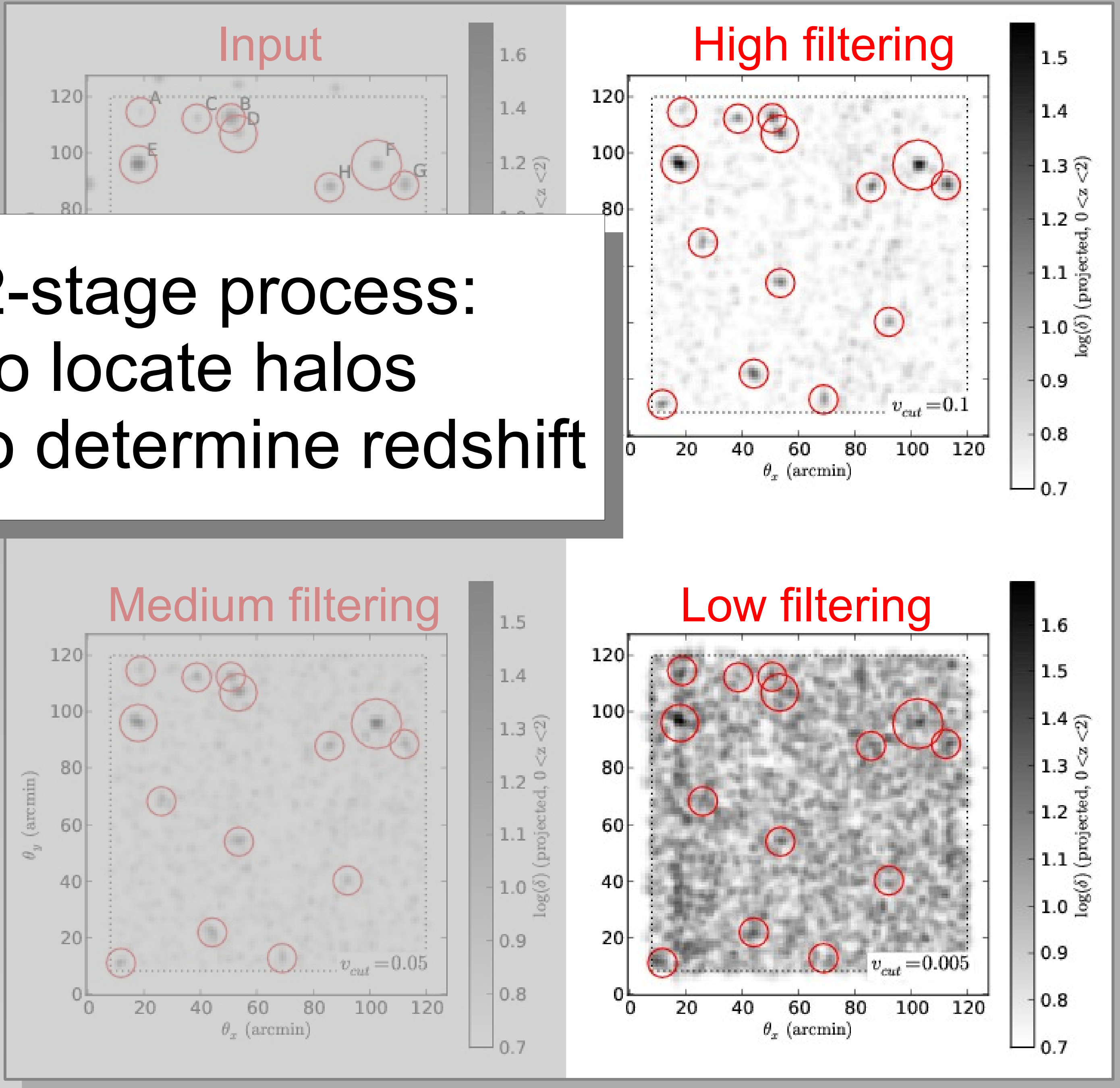
(SVD filter)





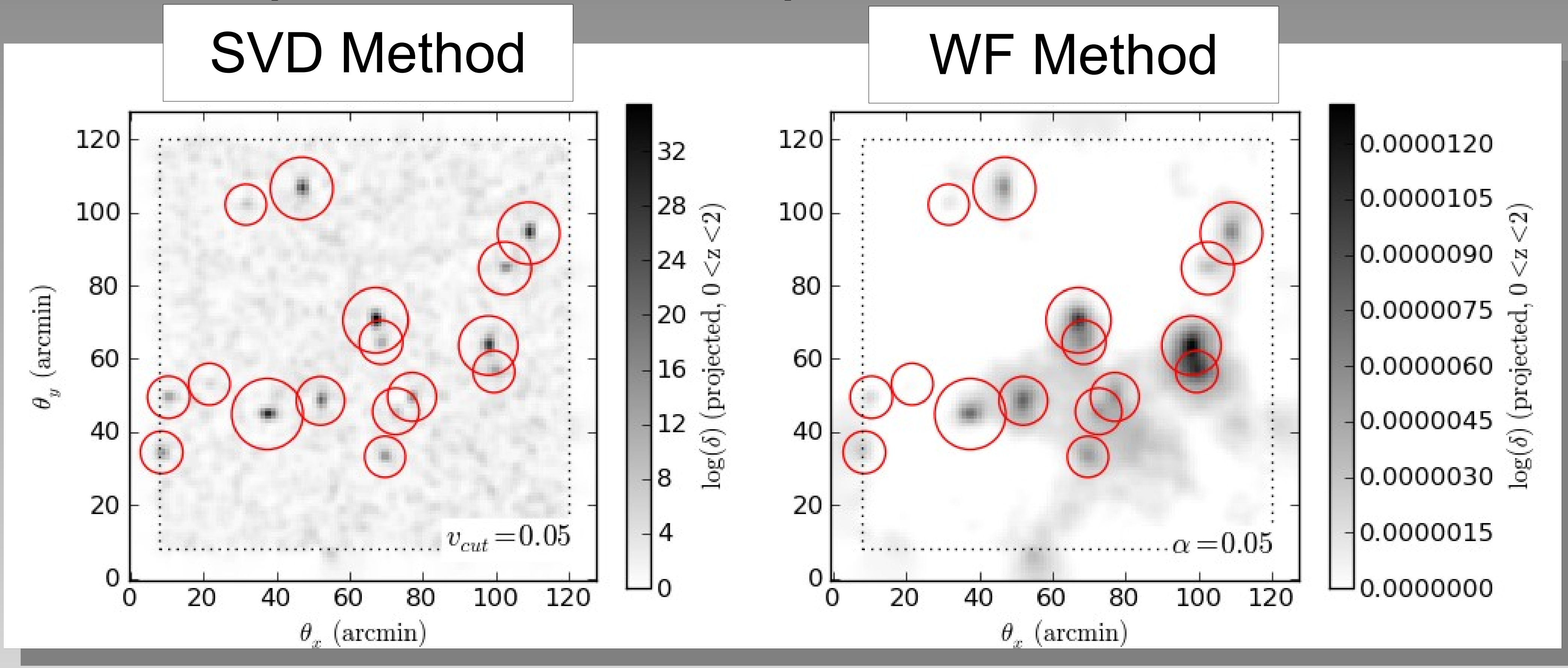
Low filtering leads to less bias and spread in redshift, but much greater noise level

Angular Resolution and Redshift Bias
(SVD filter)



This suggests a 2-stage process:
- Higher filtering to locate halos
- Lower filtering to determine redshift

Comparison for multiple lenses:



(medium filtering level for each)
SVD does better at distinguishing close pairs

Summary & Preliminary Results:

- We've developed a new non-parametric 3D mass-mapping method

- The method improves upon the Wiener filter technique for signal-suppression & angular spread

- Problems with redshift spread & bias are still unresolved – similar to Wiener filtering

- Speeds are 10-1000 times faster than Wiener filtering – applicable to future large surveys

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