Quantum Mechanics 3 2001/2002

Problem set 3

(1) [an extended combination of 2 short questions from the 1998 class exam and the 1999 degree exam]

(a) If O is a quantum-mechanical operator, what is the definition of the corresponding Hermitian conjugate operator, $O^\dagger$?

(b) Define what is meant by a Hermitian operator in quantum mechanics.

(c) Show that d/dx is not a Hermitian operator. What is its Hermitian conjugate, $(d/dx)^\dagger$?

(d) Prove that, for any two operators A & B, $(AB)^\dagger = B^\dagger A^\dagger$.

(2) [1999 degree exam short question] An infinitely deep one-dimensional potential well runs from $x = 0$ to $x = a$; the normalized energy eigenstates are $u_n(x) = (2/a)^{1/2} \sin(n\pi x/a)$, $n = 1, 2, \ldots$

A particle is placed in the left-hand half of the well, so that the wavefunction is $\psi =$ constant for $x < a/2$. If the energy of the particle is now measured, what is the probability of finding it in the ground state?

(3) The 1D potential $V = m\omega^2 x^2/2$ is that of a harmonic oscillator – e.g. a particle attached to a spring that can stretch or compress. Now suppose that the spring can only be stretched, so that this potential becomes $V = \infty$ for $x < 0$. What are the energy levels of this system?