Quantum Mechanics 3 2001/2002

Problem set 2

(1) [2000 resit paper; short question]

(a) Define what is meant by the term 'stationary state' in quantum mechanics, and hence explain the distinction between the time-dependent and time-independent Schrödinger equations.

(b) At time $t = 0$, the wavefunction of a particle is $\psi(x) = (u_1(x) + u_2(x))/\sqrt{2}$, where $u_1(x)$ and $u_2(x)$ are two solutions of the time-independent Schrödinger equation. What is the probability density for the particle at a later time t?

(2) [2000 summer paper; short question]

(a) Write down the time-independent Schrödinger equation for a particle in a one-dimensional harmonic oscillator potential, $V = m\omega^2 x^2/2$.

(b) The ground-state wave function is of the form $\psi \propto \exp(-\alpha x^2)$. Determine the constant α, and hence the ground-state energy.

(3) [1999 summer paper]

Consider a one-dimensional potential that consists of two delta functions:

$$V(x) = -|\alpha|[\delta(x-a) + \delta(x+a)].$$

(a) What can be said about the parity of the wavefunction?

(b) A single delta-potential causes a discontinuity in $d\psi/dx$ that is proportional to the value of the wavefunction at the delta-potential. By integrating the Schrödinger equation across one delta-potential, derive this boundary condition.

(c) Hence, using a graphical method, deduce the number of bound states of $V(x)$ as a function of $|\alpha|$. Show that there is always one even state, but that there is no odd state if $|\alpha| < \hbar^2/(2ma)$.